Discrepancies in the underlying zero coupon yield curve

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Financial literature and financial industry use often zero coupon yield curves as input for testing hypotheses, pricing assets or managing risk. They assume this provided data as accurate. We analyse implications of the methodology and of the sample selection criteria used to estimate the zero coupon bond yield term structure on several financial purposes. As input we consider our own spot rates estimation from GovPX bond data and three popular interest rates data sets: from the Federal Reserve Board, from the US Department of the Treasury (H15), and from Bloomberg. First, we obtain the volatility term structure using historical volatilities and Egarch volatilities. Second, we estimate correlation coefficients among forward rates. Third, we price a set of simple interest rate derivatives. We find strong evidence that the resulting zero coupon bond yield volatility estimates as well as the correlation coefficients among spot and forward rates depend significantly on the data set. We observe relevant differences in economic terms when volatilities are used to price derivatives. We also show the impacts of the yield curve choice for the results of classic term structure hypothesis tests.

Keywords: Term Structure of Interest Rates; Yield Curve Data Sets; Volatility Term Structure; Forward rates

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1.- Introduction

Most practitioners and researchers use data sets of estimated zero-coupon yield curves for computing the term structure of interest rates volatility. This relationship between zero coupon bond yield volatilities and their term to maturity is a key input of a number of financial purposes, such as calibrations of fixed income valuation models, valuations, Greek calculations, risk measurement, or design of hedging strategies.

Díaz et al. (2011) emphasize the significant impact of the chosen model for fitting the zero coupon yield curve on the volatility estimates. They estimate the yield curves using two alternative methods, the proposed by Nelson and Siegel (1987) and by Vasicek and Fong (1982), from the Spanish Treasury debt market. However, practitioners and researchers usually download the zero coupon yield curves from a database provider instead of proceeding to estimate them directly from market data. A few yield curve data sets have become popular. Some of them because they are publically available and without charge, other ones because are offered by the main financial data providers. These popular yield curve data sets consider market prices or market yields to maturity of different sets of Treasury securities and use different estimation methods. Spot rates are estimated approximating a particular functional form to a theoretical discount function that values as accurately as possible bond prices. Most papers in the literature and most practitioners seem not to be concern about these divergences between the raw material and the way the yield curves are obtained. They assume as certain and perfectly accurate one of these alternative yield curve data set without further ado.

This study investigates the effects of using alternative commonly accepted yield curve data sets on the resulting volatility of the estimated zero coupon bond yields. We also examine the statistical and economical implications on the forward rate correlations and on the pricing of simple fixed income derivatives. We examine daily data from January 1994 to December 2006 obtained from the Treasury YC estimates of the Federal Reserve Board (FRB)[1] posted on its website and commented by Gürkaynak *et al.* (2006),[2] from the YC reported by the U.S Department of the Treasury (DoT),[3] from the Bloomberg (F082) zero coupon yield curve, and from our own estimations using the Svensson method from prices reported by GovPX. These four different data sets use different estimation methods: a weighted Svensson (1994) model (FRB), a quasi-cubic hermite spline function (DoT), a piecewise linear function (F082), and a weighted and an unweighted Svensson model (our estimates). They analyze different security sets: off-the-run bonds (FRB), on-the-run bills and bonds (DoT), all the outstanding (even callable) bonds (F082), and all the traded bills and bonds (our estimates). Finally they consider different market data as input: market prices (FRB and our estimates), market yields (DoT), and generic prices (F082).

This analysis is focused on the current volatility instead of the implicit volatility. Although some interest rate volatilities can be considered as an observable variable, for

[1] This data set is called "the FRB data set" in this paper for convenience. However, the spreadsheet that can be downloaded from the FRB website contains this sentence: "Note: This is not an official Federal Reserve statistical release."
[2] It is usually cited in the literature as Federal Reserve H15 series. http://www.federalreserve.gov/econresdata/researchdata.htm
[3] http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

instance, through the quotations of contracts such as caps, caplets, floors, these volatilities are not the variable we are interested on. They are implicit volatilities, i.e. volatilities of forward interest rates instead of zero coupon bond yields.

The term structure of volatilities is a necessary input for calibrating many interest rate models and particularly the so called "volatility consistent models". Within this category we can find models such as Black, Derman and Toy model, one of the most popular tools among practitioners, or some extended versions of Hull and White model. Many financial problems, such as product valuations, risk measurements, hedging strategies, depend on spot interest rate volatilities and correlations. For instance, Value at Risk depends, above all, on the volatility of spot interest rates and their correlations.

In many topics of Macrofinance or Monetary Policy the term structure of interest rate plays a decisive role. Examples of this can be the problem of testing expectations hypothesis, the estimation of risk premium in bond markets, the ability of volatility to capture the economic uncertainty and its forecasting power with respect to the business cycle, the problem of the volatility transmission along the yield curve.

But we must be aware of the fact that when proceeding in this way we are estimating the volatility of the sum of two elements: the zero coupon bond yield itself and a "small" error term. So, the question we would like to answer is to what extent the valuations, risk measurements, hedging strategies, etc. that depends so crucially on interest rate volatilities are "contaminated" by the particular set of estimated spot rate data employed. Does it matter the method or the bond sample we (or someone else) have used to estimate the term structure of interest rates for the resulting estimates of the volatility term structure?

As far as forward rates depend of pairs of spot rates, the results of most of the test of this hypothesis will depend, in the end, on the variances and covariances among sets of spot rates with different maturities. Then, do the outcomes of these experiments depend on the actual set of spot rates eventually chosen for testing this hypothesis? Moreover, do the correlations among forward rates depend on the method used to estimate spot rates?

In this paper, we analyse to what extend the zero coupon yield curve (YC) data set affect on the resulting volatility term structure (VTS). We check whether these data sets determine the VTS even more than the own method used for fitting the VTS. We analyze statistic and economic implications of considering one among several popular YC data sets. A previous paper of Díaz et al. (2011) observes statistically significant differences in the resulting volatility term structure when alternative fitting methods and error structure assumptions are used to estimate the Spanish term structure of interest rates.[4]

We consider the aforementioned three YC data sets, FRB, DoT and F082, and our own estimations using both the unweighted and weighted versions of the Svensson method

[4] The Spanish Treasury bond market is not comparable to the US Treasury bond market in terms of size, maturity structure, number of outstanding bonds, depth, liquidity, issuance policy, on-the-run process, etc. Fitting methods that provide a good fitting in a market where on average 22 different issues are daily traded and the longest bond maturity is 15-year, give disappointing results in a market where on average 132 different issues are daily traded and new 30-year bonds are quarterly issued.

from prices of all the traded bills, bonds, and notes reported by GovPX. From these estimates of the YC, we proceed to estimate interest rate volatilities. We consider both model-free volatilities computed using a 30-day rolling window estimator and model-implied volatilities from conditional volatility models (GARCH models). A standard EGARCH(1,1) model is traditionally assumed to estimate volatility of daily interest rates (see, e.g., Longstaff and Schwartz, 1992). However, since Hamilton (1996), the EGARCH model has been widely used for analyzing the volatility of daily short term and very short term interest rates. Andersen and Benzoni (2007) have documented that an EGARCH representation for the conditional yield volatility provides a convenient and successful parsimonious model for the conditional heteroskedasticity in these series. According to the Schwarz and Akaike Information Criterion (SIC and AIC respectively), we choose the EGARCH (1, 1) model to estimate interest rate volatility.

We show statistically and economically significant differences between estimates of the term structure of interest rate volatilities depending on the YC data set and even in the structure of errors used in our own YC estimates. These differences are observed mainly in the short-term, but also in the long-term volatility. This inspection could have significant consequences for a lot of issues related to risk management in fixed income markets.

The expectations hypothesis has received a great deal of attention in the empirical literature. Although empirical researchers have frequently rejected the expectations, the empirical evidence varies from one study to the next depending on the precise implication tested, the segment of the yield curve examined or the period under study. We replicate the classical Campbell (1995) test using our different data sets. In this sense, we are exploring the expectations hypothesis by using the same model to test, the same interest rate maturities, and the same period. We obtain different results depending on the yield curve data set.

The rest of this paper is organized as follows. Section 2 describes the alternative yield curve data sets we examine. First part of the section analyses the fitting process of our own estimates using Svensson method. We describe the original data set we use, the model and the assumption about the error term variance. Second part of the section analyses the main characteristics of the external yield curve data we examine. Section 3 describes the empirical analysis that consists on the study of the impact of the methodology for estimating spot rates on the yield curve, the impact on the term structure of volatilities, the impact on the correlation of forward rates, the impact on the pricing of fixed income derivatives, and the impact on the expectations hypothesis tests. The last section includes our summary and conclusions.

2.- The alternative yield curve data sets

2.1. Our zero yield curve estimates

2.1.1. Our original data set

We obtain intraday U.S. Treasury security quotes and trades for all issues between January 1994 and December 2006 (2,864 trading days) from the GovPX database.[5] GovPX consolidates and posts real-time quotes and trades data from six of the seven major interdealer brokers (with the notable exception of Cantor Fitzgerald). Taken together, these brokers account for about two-thirds of the voice interdealer broker market. In turn, the interdealer market is approximately one half of the total market (see Fleming, 2003). Hence, while the estimated bills coverage exceeds 90% in every year of the Fleming's GovPX sample (Jan 97 – Mar 00), the availability of thirty-year bond data is limited because of the prominence of Cantor Fitzgerald at the long-maturity segment of the market. According to Mizrach and Neely (2006), voice-brokered trading volume began to decline after 1999 as electronic trading platforms (e.g., eSpeed, BrokerTec) became available. In fact, GovPX does not provide aggregate volume and transaction information from May 2001.[6] Therefore, we assume an imperceptible impact of the decline in GovPX market coverage on our estimates since we consider the midpoint prices and yields between bid and ask at 5 pm.

The GovPX data set contains snapshots of the market situation at 1 pm, 2 pm, 3 pm, 4 pm, and 5 pm. Each snapshot includes detailed individual security information such as CUSIP, coupon, maturity date, and product type (indicator of whether the security is trading when issued, on the run, or active off the run). The transaction data include the last trade time, size, and side (buy or sell), price (or yield in the case of bills), and aggregate volume (volume in millions traded from 6 pm previous day to 5 pm). The quote data include best bid and ask prices (or discount rate Actual/360 in the case of bills), and the mid price and mid yield (Actual/365).

Our initial sample relies on the information at 5 pm, i.e. last transaction taking place during "regular trading hours" (from 7:30 am to 5:00 pm Eastern Time, ET) if available, or quote data otherwise. We complement the GovPX data with official data on the dates of the last issue and of the first coupon payment, and the coupon rate of each Treasury security.[7] This information is publicly available on the U.S. Treasury Website.

To obtain a good adjustment in the short end of the yield curve, we consider all the Treasury bills. In this term to maturity segment, bills are very much more actively traded than old off-the-run notes and bonds.[8] Thus, we include only the Treasury notes and bonds that have at least one year of life remaining. Since the number of outstanding bills with terms to maturity between 6-month and 1-year declines considerably during

[5] GovPX Inc. was set up under the guidance of the Public Securities Association as a joint venture among voice brokers in 1991 to increase public access to U.S. Treasury security prices.

[6] After ICAP's purchase of GovPX in January 2005, ICAP PLC was the only broker reporting through GovPX.

[7] "Standard interest payment" field gives indirectly information to identify callable bonds and TIPS (Treasury Inflation-Protected Securities).

[8] Also, Fleming (2003) emphasizes that GovPX bill coverage is larger than bond and note coverage.

year 2000 and the 1-year Treasury bill is no longer auctioned beginning March 2001, we also consider Treasury notes and bonds with remaining maturities between 6- and 12-month from 2001.

We also apply other data filters designed to enhance data quality. First, we do not include transactions associated with "when-issued" and cash management, or trades and quotes related to callable bonds and TIPS (Treasury Inflation-Protected Securities). Second, when two or more different securities have the same maturity, we only consider trades and quotes of the youngest one, i.e. the security with the last auction date. Finally, we exclude yields that differ greatly from yields at nearby maturities.[9] In certain dates, we apply an *ad hoc* filter. We observe occasionally that deleting a single data point in the set of prices used to fit the yield curve can produce a notable shift in parameters and also in fitted yields improving notably the fitting. This phenomenon is also commented by Anderson and Sleath (1999).

Controlling for market conventions, we recalculate the price of each security in a homogeneous fashion to avoid effects of different market conventions depending on maturities and assets. Every price is valued at the trading date in an actual/actual day-count basis. In the case of Treasury bills, firstly we obtain the price at the settlement date from the last trade price if available or from the mid price between bid-ask otherwise.[10] In both cases, the GovPX reported price is a discount rate using the actual/360 basis. Secondly, we compute the yield-to-maturity as a compound interest rate using the actual/actual. Thirdly, we calculate "our" price at the trading date using the yield-to-maturity obtained in the previous step.[11] In the case of Treasury notes and bonds, the price is directly reported in the data as the last trade price or the mid price. From this price we apply the mentioned second and third steps to obtain "our" homogeneous price.[12]

2.1.2. Our term structure specification

The estimation of the spot rates consist of finding a functional form that approximates the theoretical discount function, $D(t)$, and to replicate at a given instant as accurately as possible a set of bond prices:

$$P_k = \sum_{T=T_1^k}^{T_n^k} C_T^k \cdot D(T,\bar{b}) + \varepsilon_k \qquad k=1,2,\ldots,m \qquad [1]$$

where P_k is the price of bond k, C_T^k are the cash flows (coupon and principal payments) generated by bond k, $D(T,\bar{b})$ the discount function that we want to approximate and that depends on a vector of parameters $\bar{b}$ and ε_k is an error term.

Among practitioners it is usual to estimate the yield curve from successive swap rates combined with money market data and/or coupon bond market data. The quoted swap rates can be considered as par yields for bonds. They use the simple non-parametric

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[9] These cases include interdealer brokers' posting errors like those mentioned by Fleming (2003).

[10] We do not consider the reported mid yield. This is a simple interest with actual/365 basis, except for more than 6-month remaining maturity bills which are valued using the bond equivalent yield.

[11] Note that the settlement date is in most cases a working day after the trading date.

[12] We control for the special amount of the first interest payment in just-issued securities.

bootstrapping technique to obtain spot rate estimates for certain fix maturities. Other maturities are obtained from more or less sophisticated interpolation methods. The resulting implied forward curve can be irregular e.g. curve with "sawtooth", inconsistent and sensitive to bond price variations/errors.

The term structure can be approximated using mainly two sets parametric models depending on the functional form. Models based on the discount function or spline-based approaches, such as McCulloch (1971) or Vasicek and Fong (1982), and models based on the forward curve, such as Nelson and Siegel (1987) or Svensson (1994).

These models can have different degrees of flexibility to describe the term structure of interest rates. For instance some models may have a greater ability to describe the hump so often observed in the yield curve or the behaviour of long term interest rates meanwhile others can be more rigid in the adjustment of the actual yield curve; in this case, some models may produce more or less volatile interest rate estimates in some tranches of the yield curve. If a model is too rigid to adapt to the actual shape of the yield curve it may produce in some regions of the yield curve estimates of the spot rates that fluctuate less than real spot rates do. However, a more flexible model may capture more adequately the real behaviour of interest rates.

According to the Bank of International Settlements (BIS, 2005), nine out of thirteen central banks currently use either the Nelson and Siegel (1987) or the extended version suggested by Svensson (1994) for estimating the term structure of interest rates.[13] One of the exceptions is the United States which applies a "smoothing splines" method.

Díaz et al. (2011) obtain good fitting results when they apply Nelson and Siegel (1985) and Vasicek and Fong (1982) (just one knot) in their Spanish Treasury debt sample. We initially try to use same methods from our GovPX's US Treasury debt sample. Unfortunately we discard these methods since several deficiencies are observed in most dates. It can be observed weakness in the fitting for maturities longest than fifteen years. In addition they are not able to replicate the short-term hump that is observed in most dates. These methods consider only one hump.

We apply the Svensson (1994)'s "two-hump" model which provides us more reliable fit.[14] The Svensson (1994) model can be considered as an extension of Nelson and Siegel (1985) model. Both methods are simple parametric models of the term structure of interest rates. These parsimonious approaches impose a functional form for the instantaneous forward rates. In the case of the Svensson model, the forward rates are governed by six parameters:

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[13] Svensson extension allows to capture a second hump and s-shapes, but these shapes hardly ever appear in term structures shorter than 20-years. This is the case of our sample. Also Bolder and Stréliski (1999) comparison among NS, Svensson and Super-Bell models is not conclusive. For several goodness-of-fit measures, they observe that "The Nelson-Siegel model appears to perform better than the Svensson model for the flat and inverted term structures and worse for an upward-sloping yield curve. In aggregate, they even out and show little difference." In this sense, Diebold and Li (2006) enumerate a number of authors that have proposed extensions to NS to enhance flexibility (including Svensson); however, they conclude that from the perspective of interest rate forecasting accuracy, the desirability of these generalizations is not obvious.

[14] We do not analyze results for the estimated US zero coupon yield curve using unweighted and weighted versions of Vasicek and Fong (1982) and Nelson and Siegel (1987) models. We do not consider these results in the interest of brevity. These results are available upon request from the authors.

$$f_T = \beta_0 + \beta_1 \exp\left(-\frac{T}{\tau_1}\right) + \beta_2 \frac{t}{\tau_1} \exp\left(-\frac{T}{\tau_1}\right) + \beta_3 \frac{T}{\tau_2} \exp\left(-\frac{T}{\tau_2}\right) \qquad [2]$$

where T is the term to maturity and (β_0 , β_1 , β_2 , τ_1 , β_3 , τ_2) the set of parameters to be estimated.

The two last parameters make different NS and Svensson. The latter is a more flexible approach that allows a second "hump" in the forward rate curve and also provides a better fitting of the convex shape of the yield curve in the long end.

A number of authors have interpreted the first three parameters in the NS model, β_0, β_1 and β_2, as the specific factors that drive the yield curve: level, slope, and curvature. In concrete, β_0 is related to the long-run level of interest rates and β_1 is regarded as the long-to-short-term spread. Literature pays less attention to the parameter τ_1. It is usually fixed at a prespecified value. Diebold and Li (2006) justify this simplification in terms of simplicity, convenience and "numerical trustworthiness by enabling us to replace hundreds of potentially challenging numerical optimizations with trivial least-squares regressions". Anyway, we estimate the six parameters as in the original proposal of Svensson. Our experience shows that the value of both parameter τ_1 and τ_2 are much more volatile than the values of the other four parameters, but they play a relevant role. These parameters determine inflection points and decide the position of the possible two humps in the yield curve.

NS and Svensson methods have a number of advantages over the spline methods. They are simple, they result in more stable yield curves, they require fewer data points, and they do not require finding an appropriate location of knot points which joint up a series of splines. However, spline methods allow for a much higher degree of flexibility than parametric models. Specifically, the individual curve segments can move almost independently of each other (subject to the continuity and differentiability constraints), so that separated regions of the curve are less affected by movements in nearby areas. Therefore spline approaches incorporate a wider variety of yield curve shapes than Svensson method.

2.1.3. The assumption about the error term variance

The second element in the process of estimation of zero coupon yields that can impact on the resulting volatility of interest rates is the structure of the variance of the error term ε_k of model [1].

The first estimates of the term structure of interest rates usually assumed homoskedasticity and so the model could be estimated using ordinary least-squares (OLS).

$$VAR[\varepsilon_k^2] = \sigma^2 \qquad [3]$$

But this is not a neutral assumption. In fact a small error in a short term bond price produces an important error in its yield to maturity. On the contrary, a big error on the price of a long term bond affects very slightly its yield to maturity. We should not forget that in this model the dependent variable is bond prices. And so, if we assume homoskedasticity we give the same importance to errors in the price of all bonds and that means that we are penalizing very heavily errors in the yields of long term bonds. So, to assume homoskedasticity implies forcing the adjustment in the long end of the yield curve but at the cost of relaxing the adjustment the curve for short maturities.

To correct this problems some authors suggested to penalize the valuation errors of the short term bonds and particularly it is usually suggested to correct the variance of the error term making it proportional to the bond duration that is:

$$VAR[\varepsilon_k^2] = \left(\frac{\partial P_k}{\partial Y_k}\right)^2 \cdot \sigma^2 = \left(\frac{D_k \cdot P_k}{1+Y_k}\right)^2 \cdot \sigma^2 \qquad [4]$$

where D_k is the k-bond duration, y_k its yield to maturity and P_k its price. Then the model is adjusted using generalized least-squares (GLS).

In this way we force the adjustment of the short term interest rates. But this is not free: it implies relaxing the adjustment of long term interest rates.

What we claim is that this correction of the variance of the error term affects not only to the accuracy of the estimates but also to the volatility of these estimates. Moreover, it may cause an important impact on the relative fluctuation of the estimated long and short term spot rates and so it may affect significantly the shape of the volatility term structure.

Figure 1 depicts the term structure estimations for the 18th of January 1996. The solid line represents the SV model estimate using the heteroskedatic structure. We can see that the curve describes quite well the data for short maturities. On the contrary, the dotted line represents the SV model estimate of the term structure applying the unweighted or homoskedatic scheme for the variance of the error terms. We can see that the adjustment is pretty bad for short maturities because when making this assumption the model does not care the adjustment in this side of the yield curve. It only pays attention to what happen in the other side. So we wonder if the assumption about the structure of the variance of the error term affects not only the quality of the adjustment but also the volatility of the estimated interest rates when we proceed to estimate the yield curve day after day.

[INSERT FIGURE 1]

The results have been summarised in Table 1 and Figure 2. Panel A shows the sum of squared residuals using the two methods and Panel B reports statistics of the estimated parameters. As expected, the homoskedatic estimates produce lower squared residual than the corresponding heteroskedastic estimates. In Figure 2 we picture the estimated term structures of interest rates corresponding to the first working day of June during the eleven years of our sample. These estimates were obtained using the Svensson model and assuming heteroskedasticity.

[INSERT TABLE 1 AND FIGURE 2]

2.2- The external zero yield curve data sets

We compile zero coupon yield curves from three external data sets. They use different estimation methods, security sets, and bond prices. First, we examine the Treasury YC estimates of the Federal Reserve Board (FRB) posted on its website and commented by Gürkaynak *et al.* (2006). They use a weighted version of the Svensson (1994) method from prices of all the outstanding off-the-run bonds. Among other securities, they exclude in their estimation all Treasury bills, and the on-the-run and the "first-off-the-run" issues of bond and notes.

Second, we analyze the yield curve reported by the U.S Department of the Treasury (DoT). Treasury does not publish historical data of these rates but they can be downloaded as H.15 in the Federal Reserve Statistical Release. They use a quasi-cubic hermite spline function that passes exactly through the yields on the chosen securities as YC method.[15] Thus DoT does not estimate a zero-coupon term structure since they just obtain a yield curve (a relation between yields to maturity and terms to maturity). They consider it as a "par curve" since the on-the-run securities typically trade close to par. No details about the concrete used functions are reported by DoT. As inputs they use the yields for the on-the-run securities. They include four maturities of most recently auctioned bills (4-, 13-, 26-, and 52-week), six maturities of just-issued bonds and notes (2-, 3-, 5-, 7-, 10-, and 30-year), plus the composite rate in the 20-year maturity range.

Third, we consider the Bloomberg (F082) zero coupon yield curve. They use a piecewise linear function from Bloomberg generic prices of all the outstanding Treasury bonds. No details about the concrete used functions are reported by Bloomberg. They estimate the zero coupon yield curve which they use to generate "Bloomberg fair value" curves for pricing most bonds that are traded over-the-counter or are illiquid bonds.[16]

3. Empirical analysis

3.1. The impact of the methodology for estimating spot rates on the yield curve

3.1.1. The flexibility of the model

We compare our Svensson's estimates with the reported by three popular data sets: the US Department of Treasury (DoT), the Federal Reserve Board (FRB), and Bloomberg (F082). Each one uses a different fitting technique and a different bond set.

Figure 3 depicts the original yields to maturity of the traded bonds, notes, and bills in the US Treasury market on July 5th 2006. Also the different term structures of interest rates are represented. The vertical axis represents interest rates and the horizontal axis the term to maturity of the securities traded in that market. The dots correspond to the yields to maturity of these securities. The lines represent estimations of the term

[15] http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/yieldmethod.aspx
[16] Bloomberg explains that a piecewise model contains more points than parameterized smooth curve. Thus they improve the fit of the yield curve. However, they recognize that this function could result in unstrippable zero curves and negative forward rates as well. See M. Lee, International Bond Market Conference 2007, Taipei.

structure using alternative models. The zero coupon interest rates are fitted using unweighted and weighted versions of Svensson from our bond price data set (GovPX) and the yield curves reported by the Department of Treasury (DoT), the Federal Reserve Board (FRB), and Bloomberg (F082).

[INSERT FIGURE 3]

It is no easy to find a function that can capture the hump that can be observed quite often in the US market. For instance, the DoT curve shows clearly mispriced zero-coupon interest rates for maturities between 2- and 7-year. Even the FRB curve obtained using weighted Svensson has not the desired characteristics for several reasons. First, the short-end of the curve is not fitted (the curve provides very short-term interest rates around 5.47% and the observed ones are around 4.85%). Second, the convexity problem appears in the long-end of the term structure. Most flexible methods usually present this problem which prevents that these long-term zero coupon interest rates meet the requirements to supply credible forward interest rates. The forward rates are very sensitive to the shape of the yield curve particularly in the very long end. Zero coupon yield curve must be asymptotically flat in order to provide a desirable flat forward rate curve. Gürkaynak *et al.* (2006) emphasize that convexity makes it difficult for fitting the entire term structure, especially those securities with maturities of twenty years or more.[17] They maintain that convexity tends to pull down the yields on longer-term securities, giving the yield curve a concave shape at longer maturities. This is the case of DoT, FRB and F082 yield curves in this particular date.[18]

3.1.2. The importance of the actual data set

In this section we point out that assets eventually selected in the estimation of the term structure of interest rates can have a significant impact on the resulting volatility estimates of spot rates.

In the Treasury market are traded securities with important differences, such as remaining maturity, different degrees of liquidity, tax premia, or embedded optionality and other features.

The BIS (2005) technical report comments the importance of the maturity spectrum. Most central banks exclude part of the maturity spectrum for which debt instruments are available. For instance, only the interval from one to 10 years is considered by certain central banks. In modelling the short-end of the term structure, this decision concerns mostly the choice of the types of short-term instruments regarded to be the most suitable and the minimum remaining term to maturity allowed in the estimation.

In this sense, the Treasury debt security data sets used in the estimation of the four zero coupon yield curve sources analyzed in this paper are quite different. Considering securities with remaining maturities shortest than one year, we include only the traded

[17] Convexity is understood as that obtained from the second-order approximation of the change in the log price of the bond.

[18] It is not possible to replicate the observed 30-year bond price from the spot rates obtained from H15 and F082 yield curves since the 30-year spot rate in both cases is lower than the yield to maturity of the bond.

bills reported in the GovPX's US Treasury debt sample;[19] the FRB estimates only include the "second-off-the-run" bonds or older, i.e. they exclude bills and the on-the-run bond and the "first-off-the-run" bond; the DoT uses as input the four most recently auctioned bills (4-, 13-, 26-, and 52-week); and Bloomberg considers all the outstanding Treasury bonds.

Liquidity may have an important impact on prices. Sarig and Warga (1989) and Warga (1992) suggest that younger bonds are usually traded more frequently. Warga (1992) uses an auction status dummy variable that indicates whether or not an issue is "on-the-run" (i.e., the most recently issued security of a particular maturity). Amihud and Mendelson (1991) observe that bonds approaching maturity are significantly less liquid since they are "locked away" in investors' portfolios. Goldreich et al. (2005) emphasize expected liquidity over the full life of the issue –not just the current level of any liquidity measure– as the most relevant theoretical constructs for valuing bond liquidity.

Usually when a bond is just issued it concentrates the most of the trading volume as most investors and fund managers are trying to allocate or distribute this new asset in their portfolios or within their clients. But as this bond becomes seasoned and above all when new references are issued the trading volume decreases dramatically and so does its liquidity. And this seems to have an important impact on bond prices.

In our zero-coupon yield curve estimates we include all the traded bills plus the traded bonds and notes with at least one year of life remaining reported in the GovPX's US Treasury debt sample. The FRB estimates only include second-off-the-run or older bonds with more than three-month to maturity. They exclude quotes of all securities with less than three months to maturity, all Treasury bills, all twenty-year bonds since 1996, the on-the-run and the first-on-the-run bonds and "other issues that we judgmentally exclude on an *ad hoc* basis". The DoT uses as input bid-side yields from the four most recently auctioned bills (4-, 13-, 26-, and 52-week), the six maturities of on-the-run bonds and notes (2-, 3-, 5-, 7-, 10-, and 30-year), plus the composite rate in the 20-year maturity range. To fit the F082 series, Bloomberg consider all the outstanding Treasury bonds, i.e. callable or not callable, and traded or not traded during the day. No bills are included.

Some authors use actual transaction prices, some other quoted prices (mid bid-ask, bid or the ask prices), and some other yields to maturity. We consider the information at 5 pm reported by GovPX, i.e. last transaction price taking place during "regular trading hours" (from 7:30 am to 5:00 pm Eastern Time, ET) if available, or quote data otherwise (mid bid-ask quotes from 2001). The DoT uses as inputs the "close of business" bid yields for the on-the-run securities. They use composites of off-the-run bonds in the 20-year range reflecting market yields available in that time tranche. Gürkaynak *et al.* (2006) comment that they use end-of-day prices for their FRB estimates but they do not specify what kind of prices are. Finally Bloomberg considers "Bloomberg generic" (BGN) prices. They are obtained as the simple average price of all kinds of prices, including indicative prices and executable prices, quoted by their price contributors over a specified time window.

[19] We also consider Treasury notes and bonds with remaining maturities between 6- and 12-month from 2001.

BIS (2005) emphasizes that premia induced by tax regulations are notoriously difficult to deal with. Several postures can be adopted: attempting to remove tax-premia from the observed prices before they are used in estimations, excluding instruments with distorted prices from the data set, or ignoring this problem altogether. We assume that the four data sets we analyze adopt the last option.

During the sample period, a group of old 30-year callable bonds is outstanding. Market prices the optionality and these bonds are traded at extremely high yields to maturity.[20] Gürkaynak *et al.* (2006) explicitly mention that they exclude all securities with option-like features. We also omit these bonds. They cannot be included in the DoT sample since they only consider on-the-run bonds. Thus, only Bloomberg takes into account these bonds.

The upper panel of Figure 4 depicts the observed yields to maturity for the traded securities on September 9th 1999. It can be seen the gap in terms of yield to maturity between the on-the-run bonds and the off-the-run bonds. So when adjusting the yield curve we have to decide if we include all the bonds, only the most liquid ones or just the opposite. But depending on this decision we are estimating, in fact, different interest rates: the spot rates corresponding to average market liquidity level, the spot rates of the most liquid references or the spot rates of seasoned bonds. The level of these interest rates should be different, but probably the volatility is different too.

[INSERT FIGURE 4]

Lower panel of Figure 4 and also Figure 3 illustrate some of the differences that affect both the level and the shape of the term structure. We can outline at least three points. First, the differences in level correspond mainly to the estimations of the Department of Treasury. Second, we can appreciate important differences in the shapes of the other three estimations although they all used Svensson's model. The Federal Reserve produces estimates with a concave shape in the long end of the yield curve: this may have a very important impact on the forward rates with longer maturities. Third, the unweighted scheme tends to produce a different adjustment in the short end of the yield curve.

At first glance these divergences in the estimation method and in the sample composition prognosticate relevant differences in the estimated yield curve and hence in the estimated volatility term structure. However, most papers in the literature and most practitioners seem not to be concern about that. They assume as certain and accurate one of these alternative yield curve data set without further ado. We wonder if all these elements can have a statistically and economically significant impact on the resulting volatility of the estimated zero coupon bond yields and this is what we have tried to answer.

[20] In the date shown in Figure 3, 11 callable bond issues with remaining maturities between 10 and 15 years were traded at yields to maturity between 7.5% and 8.5%.

3.2. The impact on the term structure of volatilities

3.2.1. Volatility estimates

From the three external yield curve data sets and our two yield curve data sets corresponding to our Svensson estimates, we extract 27 different spot rates with maturities ranging from one week up to 30 years.

From these data sets we use two alternative methods to estimate the volatility term structure (VTS). First, we calculate simple standard deviation measures using 30-day rolling windows from log-difference of the value of the spot rates. We call the resulting annualized volatilities as "historical volatilities". Second, we considered different specifications of the well known family of the conditional volatility models. Finally we choose the EGARCH(1,1) model proposed by Nelson (1991) which allows for asymmetric impacts of the innovations. Table 2 summarizes some statistical results of the VTS estimations

[INSERT TABLE 2]

In order to illustrate the results for different assumptions about the variance of the error term, Figure 5 depicts the term structure of volatilities obtained from our Svensson spot rates using the unweighted version in the panel A (homoskedastic assumption) and the weighted version in the panel B (heteroskedastic one) for the first working day of June during the sample period. There is a great diversity of shapes that the volatility term structure has taken during the sample period. We can see increasing curves, humped curves, double humped curves … It is evident that single factor models of the term structure can hardly capture this variety of profiles and shapes of the volatility.

[INSERT FIGURE 5]

Although these volatilities term structures have been chosen randomly, we can see that the shape of the term structure changes significantly depending of the weighting scheme we had chosen to estimate the term structure of interest rates.

We can also observe that humps shifted towards the left and also for very short maturities, the weighted scheme produced estimates of spot rates with a higher volatility. All this is due to the fact that using the heteroskedastic assumption forces the adjustment in the short end of the curve. On the contrary, the volatility estimates of very long term rates are higher when the weighted scheme is introduced.

The differences in the VTS obtaining for the alternative data sets are particularly relevant in some dates. Figure 6 depicts an example for the July 3rd 2006. We can observe differences in shapes and in levels during the entire maturity spectrum but especially relevant in the short end of the curves.

[INSERT FIGURE 6]

3.2.2. Tests of differences in the volatility estimates

To test if the observed differences in the VTS are significant from a statistical point of view we applied a sign test. This test allows checking if two alternative models produce significant differences in the resulting spot rate volatilities.

This test assumes as null hypothesis that given two alternative models to estimate zero coupon bond yields, the probability that one of them produces a higher volatility estimate than the other for a given day is 50 %. Thus the null hypotheses assumes that the method used to estimate the spot rates do not produce significant differences in the resulting zero coupon bond yield volatilities.

As we used a 30 day window to estimate the volatilities we selected one out of thirty estimates from our 2717 daily volatility estimates in order to avoid autocorrelation problems. Eventually, we had 91 independent volatility estimates for each maturity.

Under the null hypothesis the number of times that one method produces a higher volatility estimate than the other (x) is distributed according to a binomial random variable with parameters N=91 and p=0,5. As N is big enough we eventually assumed that X can be approximated by a normal distribution with mean $N \cdot p$ and variance $N \cdot p \cdot (1-p)$, that is $X \sim N(55{,}5;\ 27{,}75)$. The results are summarised in Table 3.

[INSERT TABLE 3]

This table shows which method (compared by pairs) produces lower volatility estimates. Panel A and panel B indicate these results for both alternative volatility specifications: historical volatility and EGARCH model respectively.

Looking at Figure 6 and Table 3 there are various issues that should be highlighted. The first one is that alternative models produce a significantly different volatility term structure, differences that seem to affect all maturities. Particularly, the most rigid model seems to provide the less volatile zero coupon rates above all for the shortest and longest maturities.

On the whole, we can state that differences are quite important. The FRB estimates produce significantly lower spot rate volatility than other estimates including those of the DoT (except for very short maturities). At the same time, the DoT estimates seem to produce less volatile spot rates than our estimates although these results are not so clear when the GARCH model is used to estimate volatilities.

3.3. The impact on the correlation of forward rates

Forward rates play a key role in many financial issues, such as the implementation of interest rates models as Heath, Jarrow and Morton, or in many product valuations where correlations among forward rates are crucial (for instance in swaptions), or for testing the Expectations Hypothesis.

But at the same time forward rates can be very sensitive to the method employed in estimating the yield curve and so the correlations among forward rates . . . We have to highlight the fact that the forward rates are very sensitive to the shape of the yield curve particularly in the very long end.

To test if the way used to estimate the term structure of interest rates have a significant impact on correlations among forward rates we have proceed to estimate them.

First we have estimated the correlations using the two sets of our zero coupon bond estimates from GovPx data base: Svensson models using both the homoskedastic and the heteroskedastic assumptions.

We can observe that these correlation coefficient estimates differ significantly from one model to another. Analyzing the results there are some patterns that can be pointed out. The first one is that weighted schemes produce lower correlation coefficients. This is illustrated in Figure 7 where we have represented the evolution of the correlation coefficient between two year and five year forward rates (with six month tenor) corresponding to Svensson model using the homoskedastic (or unweighted) and the heteroskedastic (or weighted) assumption about the error term.

[INSERT FIGURE 7]

This result that can be generalized to all maturities and it is a consequence of the fact that using the weighted scheme forces the adjustment of the yield curve in such a way that affects the entire curve, making the resulting changes in the yield curve less "linear". We can also see that these differences were higher during the second half of the sample.

In the next pictures we have illustrated the evolution of the correlation coefficient between different pairs of forward rates using the four alternative estimates of the yield curve.

The Figure 8 represents the correlations using again a 30 days window between six month spot rates and the one year forward rates with a six month tenor.

[INSERT FIGURE 8]

We can see that differences are huge during the first half of the sample period. The FRB estimates produced the most stable and highest estimates. Some of these differences may come from the fact that FRB dropped from the sample those assets with a maturity lower than three moths. That means that they did not care too much about what happened in the very short end of the yield curve. However, if we have a look at the yield curve during the first years of the sample the term structure had a very steep slope in the shortest maturities. And forward rates are very sensitive to the slope of the yield curve. So the FRB estimation eliminates or at least softens some of the sharp changes and movements that forward rates experienced during this part of the sample period.

On the contrary Figure 9 shows as the correlations among mid maturity forward rates the differences are not so severe. The only one with a different behavior corresponds to our estimates using unweighted scheme.

[INSERT FIGURE 9]

But the most astonishing results were obtained when estimating correlations between medium and long term forward rates. It must be taken into account that forward rates in the very long end of the yield curve are very sensitive to the slope of the zero coupon yield curve. As we can see in figure 10, the differences are dramatic.

[INSERT FIGURE 10]

As we can see the estimates of the FRB and the F082 indicate that 10 year forward rate and 30 year forward rate are nearly linearly independent meanwhile our estimates would suggest just the opposite, that they are practically linearly dependent particularly if we use Svensson with the homoskedastic scheme. The method applied by the Department of the Treasury produces forward rates with behaviour in the mid point between our estimates and those of the Federal Reserve and Bloomberg. All these differences are corroborated using a sign test.

In Table 4 we present the outcomes of applying the sign test to corroborate if the differences between the VTS extracted from alternative yield curve data sets are significant from a statistical point of view. We observe statically significant differences in almost all the compared pairs and maturities.

[INSERT TABLE 4]

These results confirm that weighted Svensson model produce lower correlations than the unweighted version for most maturities. And the second result that can be highlighted is that Bloomberg yield curve data set (F082) produces the lowest correlation coefficients compared with the other alternative data sets.

3.4. The impact on the pricing of fixed income derivatives

In previous sections we observe statistical differences in the VTS and correlation coefficients for interest rates of different maturities. This fact implies bad news for academics and researchers that assume the yield curve that they use as perfectly accurate. They are not concern about intrinsic differences between alternative popular yield curve data sets.

These differences should have an impact of economic terms when these alternative data sets are used as input for calibrations of fixed income valuation models, valuations, Greek calculations, risk measurement, capital requirements, or design of hedging strategies. But, is this economic impact large enough to be concerned about that? It is possible that a significant difference in statistical terms has an irrelevant impact in economic terms. In this case, this fact would be good news for practitioners that use indiscriminately these yield curve data sets. Otherwise they should pay more attention to the data set they use and should examine what the more accurate data set for their concrete valuation purposes is.

In this section, we propose an example of pricing simple fixed income derivatives in order to quantify this impact in terms of dollars. We estimate call prices and call deltas writing in theoretical callable bonds obtained from Black, Derman and Toy (1990) estimations using zero coupon rates and volatilities from the five considered datasets.

This model is considered as the most popular used by the industry. The models fit exactly the current term structure of interest rates and the current term structure of spot rate volatilities.

The call price is the difference between a straight Treasury bond price and a callable Treasury bond price which includes one or two at par call options. We assume that both securities are 7% semiannual coupon bonds. We build the theoretical straight bond by discounting the cash flows from the appropriate spot rates for each maturity. The callable bond price is computed using the BDT model from the term structure of interest rates and the VTS for each date and each data set. The call delta is the ratio between the change in price of the call option and the change in price of the underlying bond.

Tables 5 and 6 summarize the results. For each model, we obtain the average value of call prices and call deltas for the 33 BDT estimations corresponding to the first working day of March, July, and November from 1996 to 2006. These averages are compared to the average values for the 5 datasets.

[INSERT TABLES 5 AND 6]

Results show relevant differences in economic terms. For instance, the average call price using F082 dataset (Bloomberg) zero coupon rates and standard deviation volatilities for a Treasury bond with USD 100 principal, a remaining term to maturity of 30 years, and two call options at par on 20 and on 25 years is on average USD7.42 during the 33 BDT estimations. The average call price for the 5 models is USD6.15. Thus, the variation respect the average is (7.4241–6.1469)/6.1469 = 20.78% or USD1.28. In this sense, the Bloomberg dataset gives a call price 39.41% higher than the price obtained from the unweighted Svensson yield curve estimates obtained from the GovPX bond dataset, i.e. a difference of USD 2.42 per USD 100 principal. In the case of using EGARCH(1,1) volatilities, this difference between the call price that we obtain from Bloomberg dataset and that obtained from the weighted Svensson yield curve is 45.49%, i.e. a difference of USD 2.66 per USD 100 principal.

3.5. The impact on the expectations hypothesis

Finally, Tables 7 and 8 report the sensitivities of the results from Campbell's (1995) tests of classic term structure hypotheses tests.

Work-in-progress

4.- Conclusions

Alternative yield curve data sets are usually assumed as perfectly correct when they are use as input for a number of financial purposes. Practitioners and researchers do not question the accuracy of the yield curve that they obtain from data providers.

In this paper we examine three popular data sets and our own estimate of the zero coupon bond yields from the Treasury market. We analyse the differences in terms of fitting methodology, in the considered sample of securities, and in the price or yield used as input. We observe statistically significant differences appear in the volatilities and correlations among the resulting series of spot and forward rates with different maturities particularly in the short and long ends of the range of maturities. These differences have a dramatic impact on the correlations among forward rates with different maturities. Finally, we observe relevant differences in economic terms when we apply the yield curves and the volatility term structure to price a very simple fixed income derivative.

These observed differences imply bad news for academics and researchers that assume the yield curve that they use as perfectly accurate. They are not concern about intrinsic differences between alternative popular yield curve data sets. These differences are also bad news for practitioners that use indiscriminately these yield curve data sets. They should pay more attention to the data set they use and should examine what the more accurate data set for their concrete valuation purposes is.

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Figure 1.- The importance of the variance of the error term

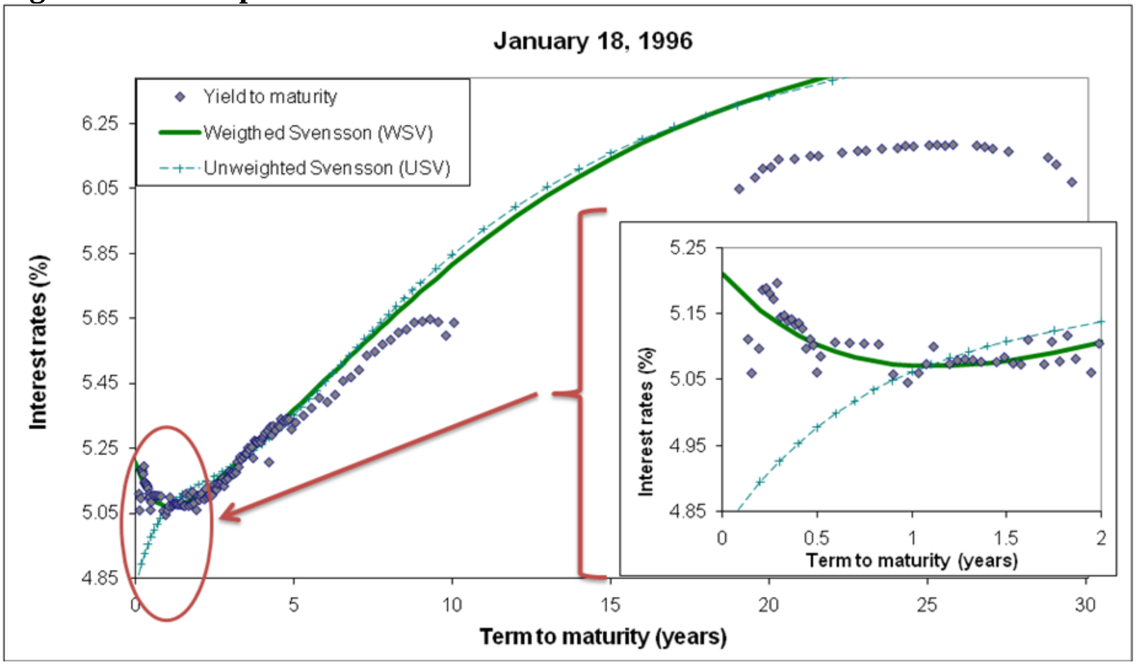


Figure 2.- Term structure of interest the first working day of June during the sample period. (01.96-12.06) .

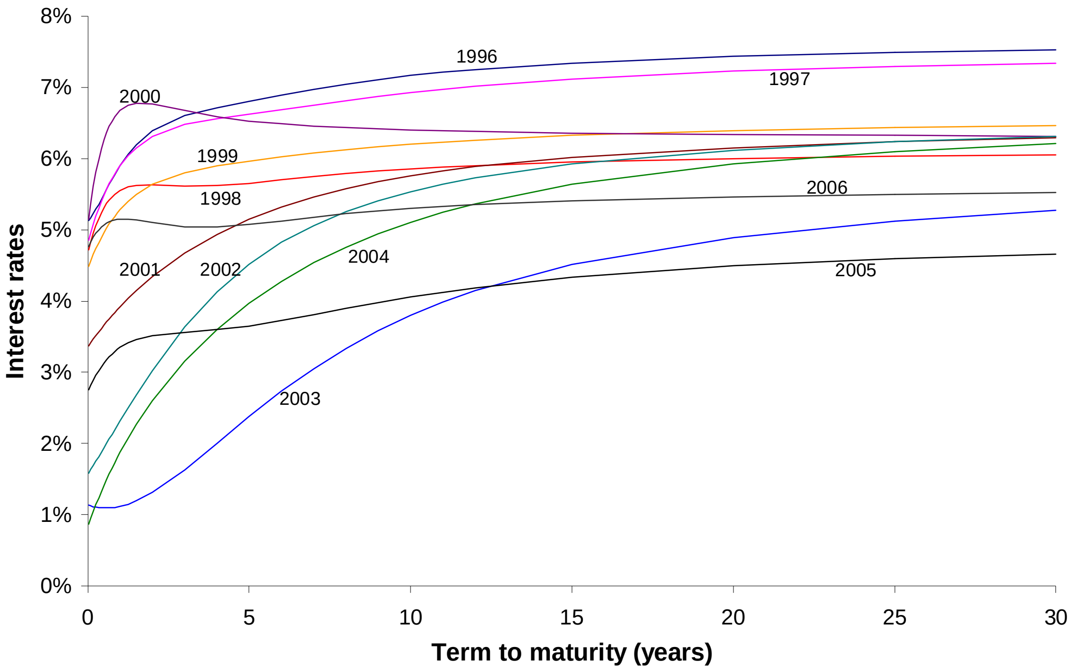
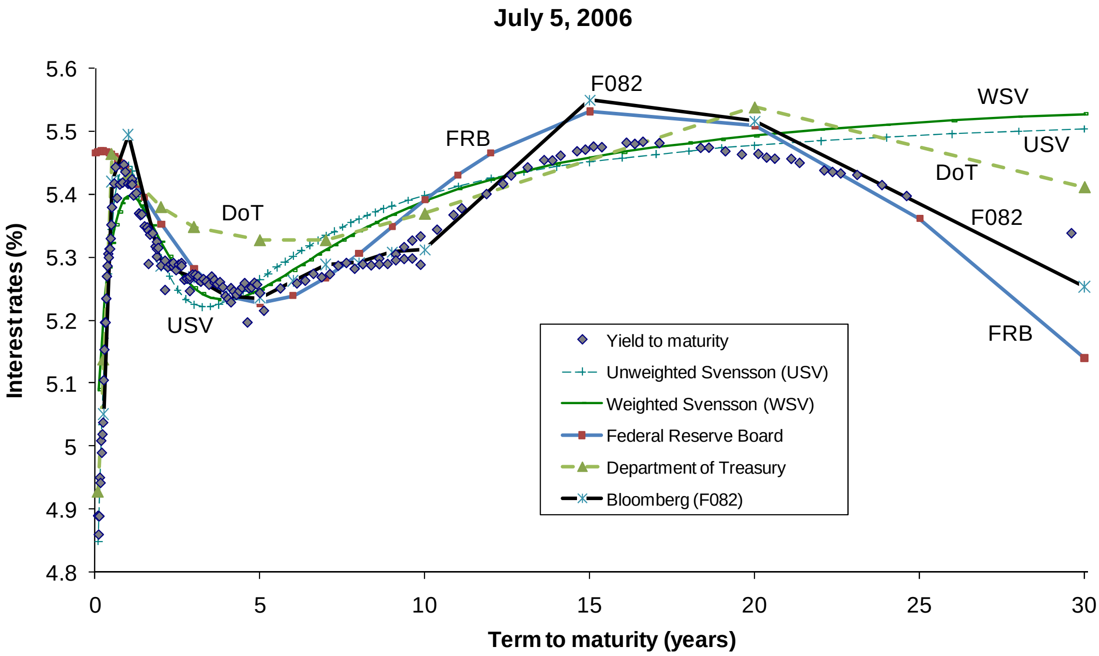
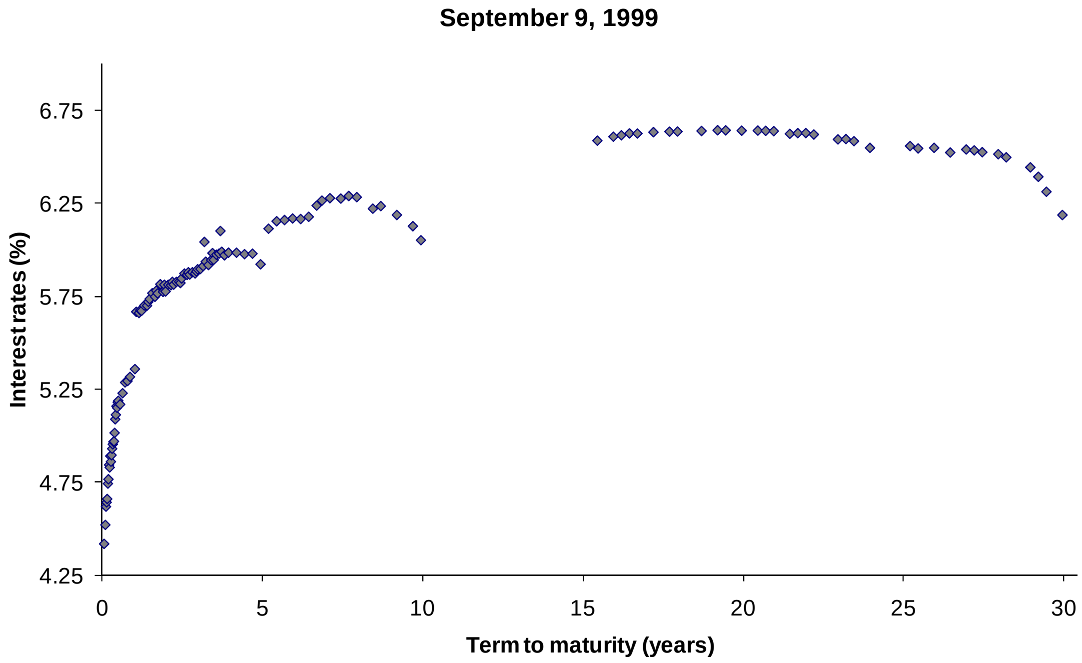


Figure 3.- Alternative estimations of the term structure of interest using models with different degrees of flexibility



Note: the points represent the yields to maturity of all the non-callable traded Treasury securities.

Figure 4.- The impact of liquidity on yields to maturity



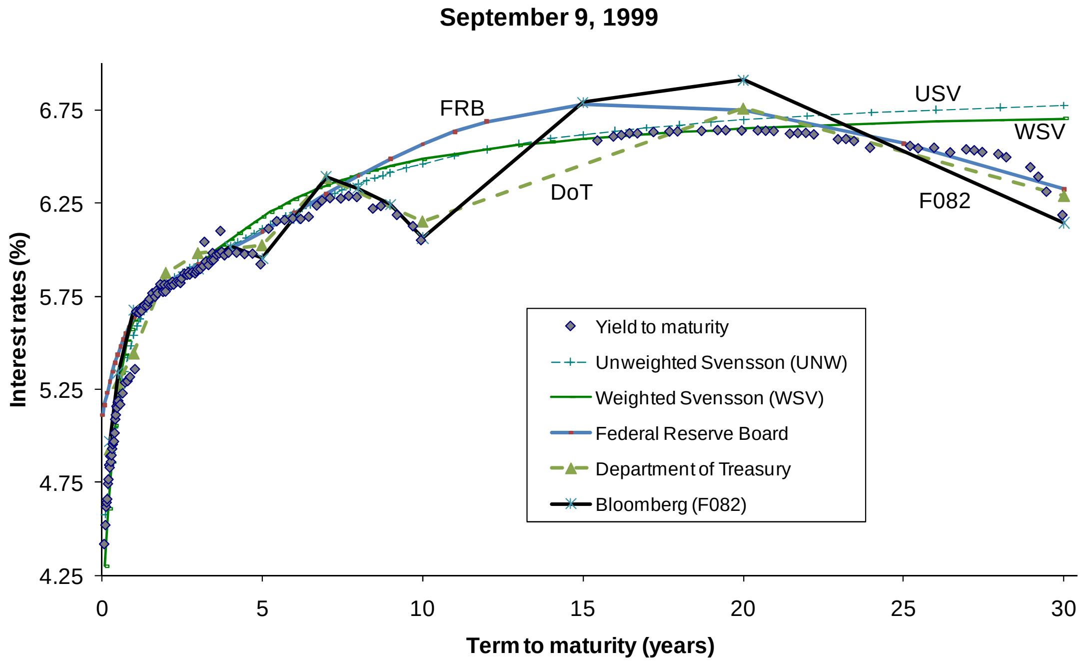
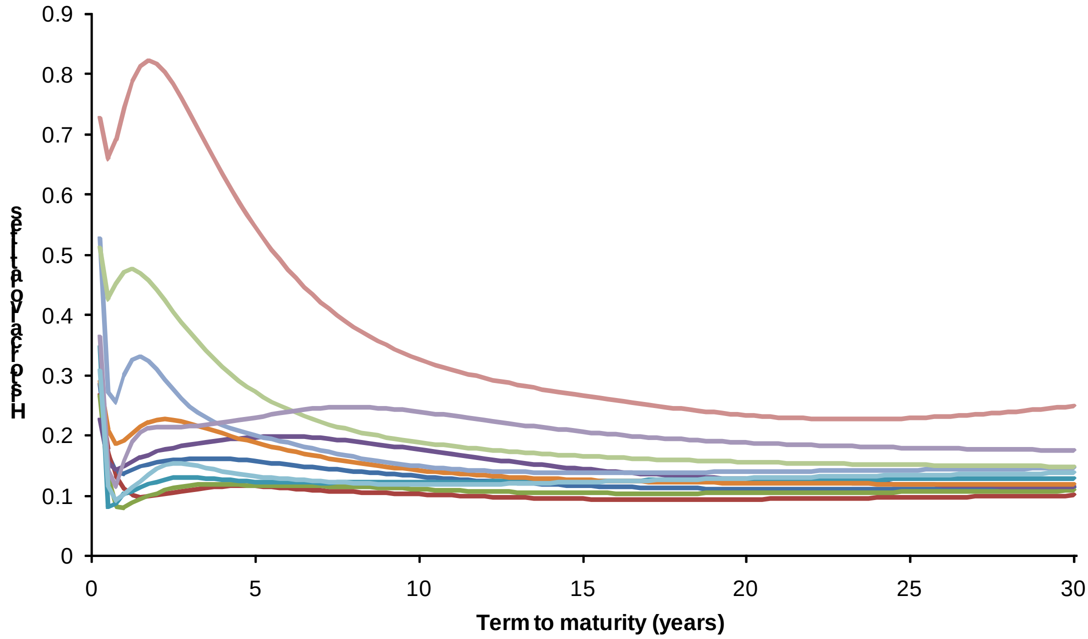


Figure 5.- The impact of the assumption about the variance of the error term
Panel A.- Svensson model with homoskedastic error. VTS corresponding to the first working day of June during the sample period



Panel B.- Svensson model with heteroskedastic error. VTS corresponding to the first working day of June during the sample period

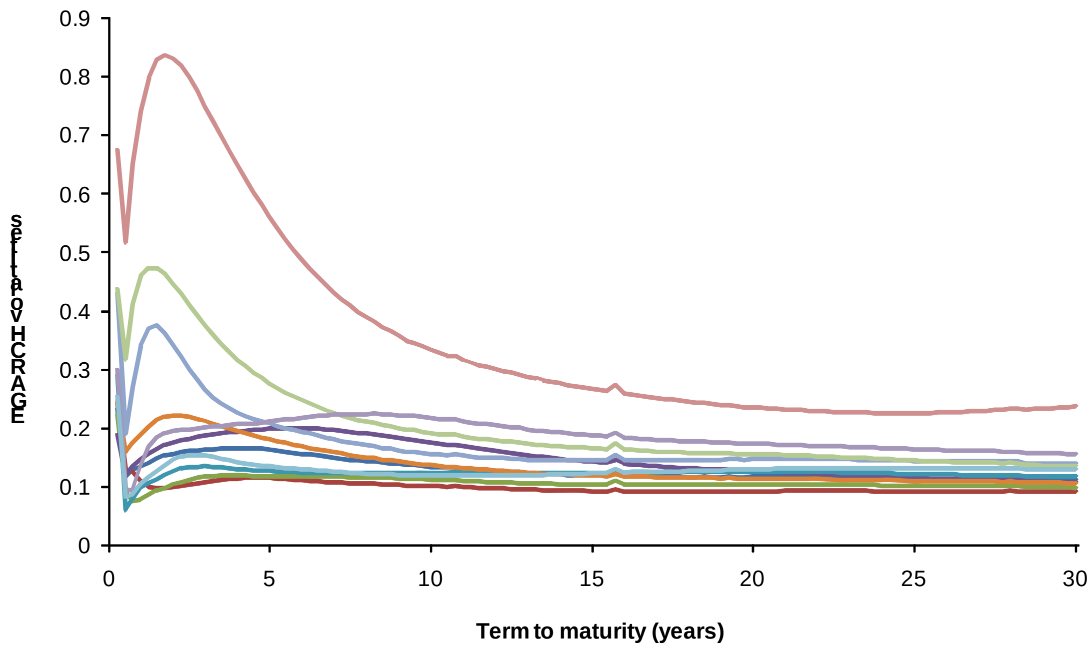


Figure 6.- Historical volatility term structure estimates from alternative yield curve data sets (July 3, 2006)

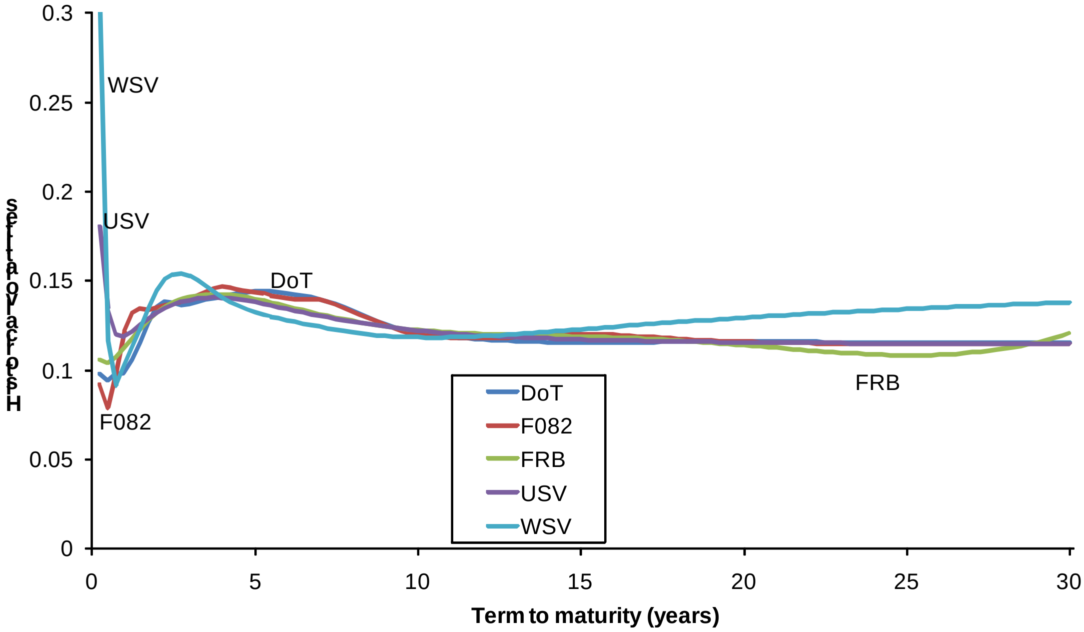


Figure 7.- The impact of the assumption about the error term. Correlations between the 2-year-ahead 6-month forward rate and 5-year-ahead 6-month forward rate using Svensson's model and two alternative assumptions about the variance of the error term

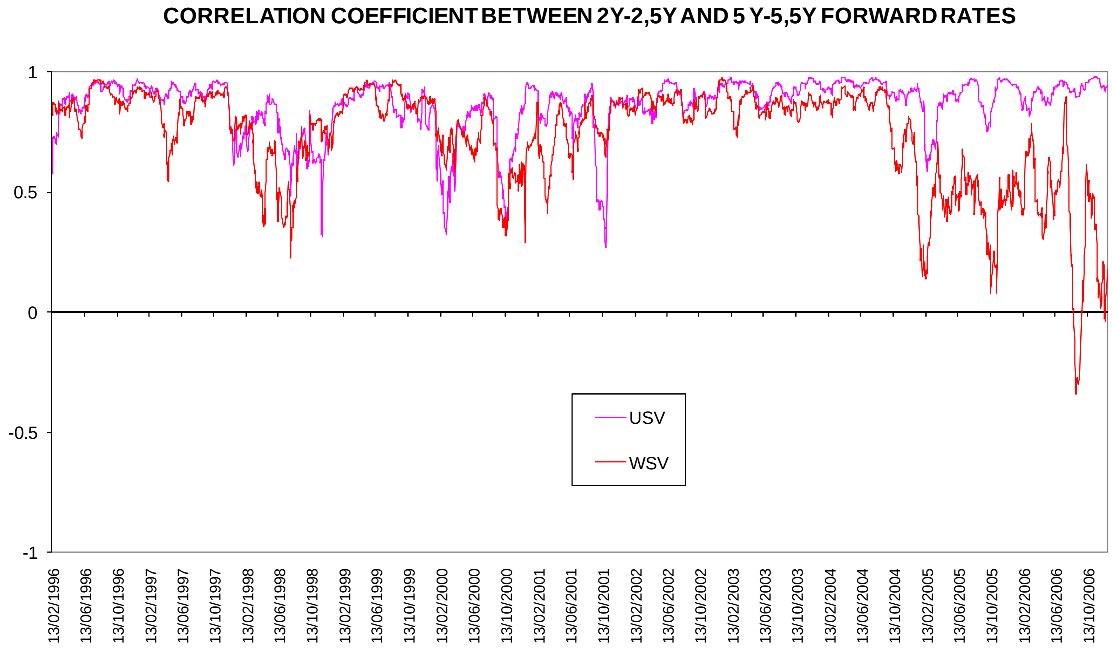


Figure 8.- Correlations between 6-month spot rate and 1-year-ahead 6-month forward rate

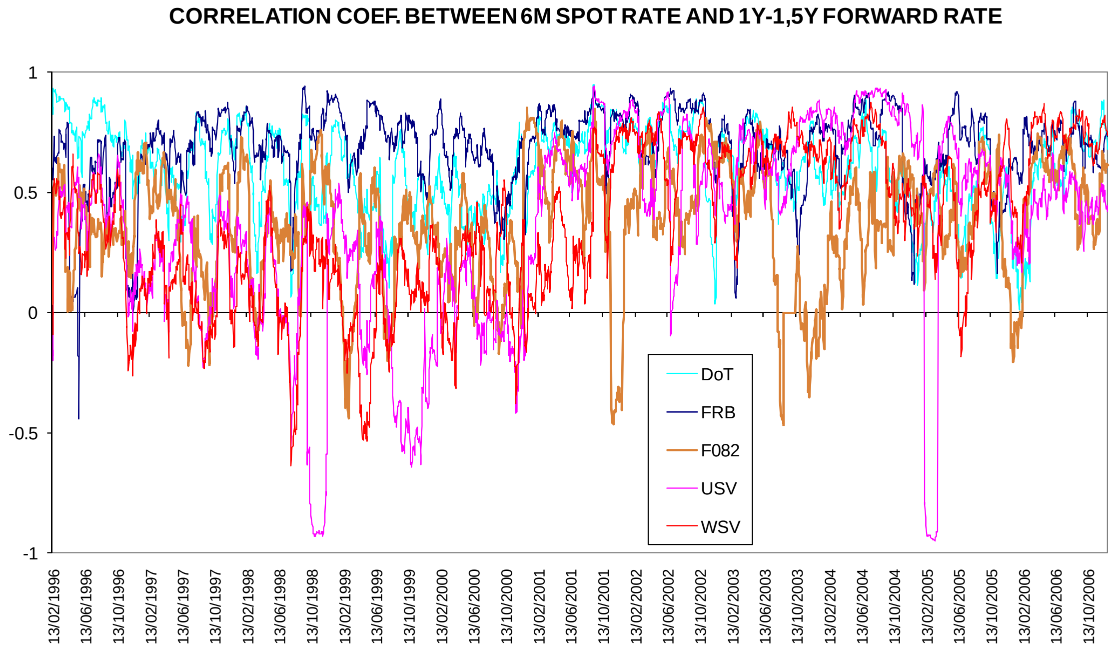


Figure 9.- Correlations between the 2-year-ahead 6-month forward rate and 5-year-ahead 6-month forward rate

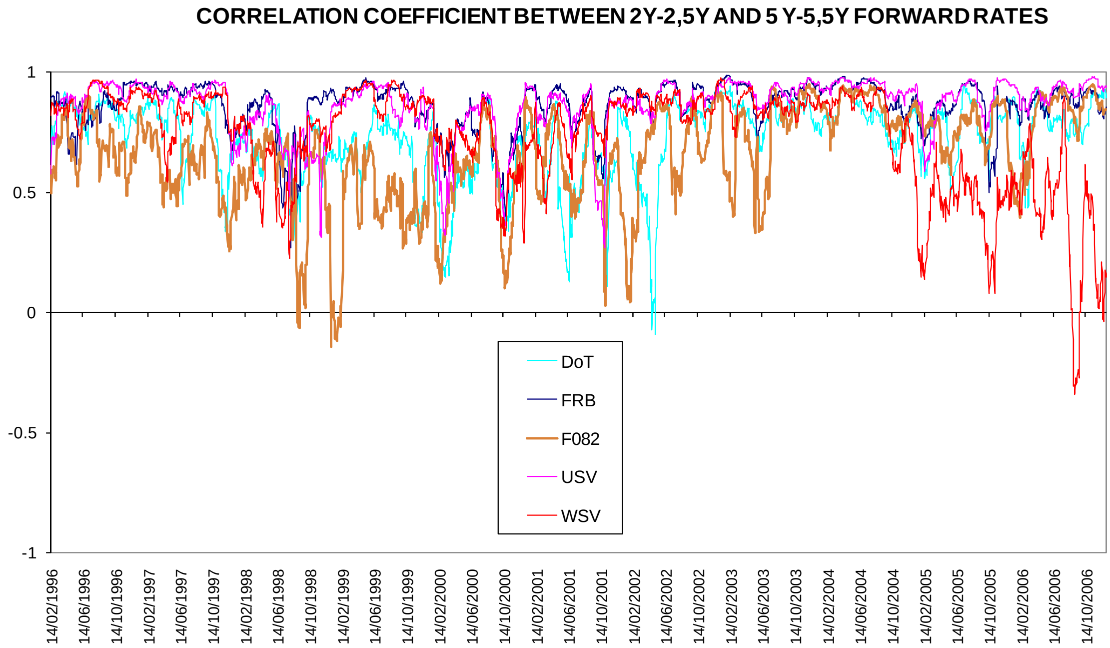


Figure 10.- Correlations between the 10-year-ahead 6-month forward rate and 30-year-ahead 6-month forward rate

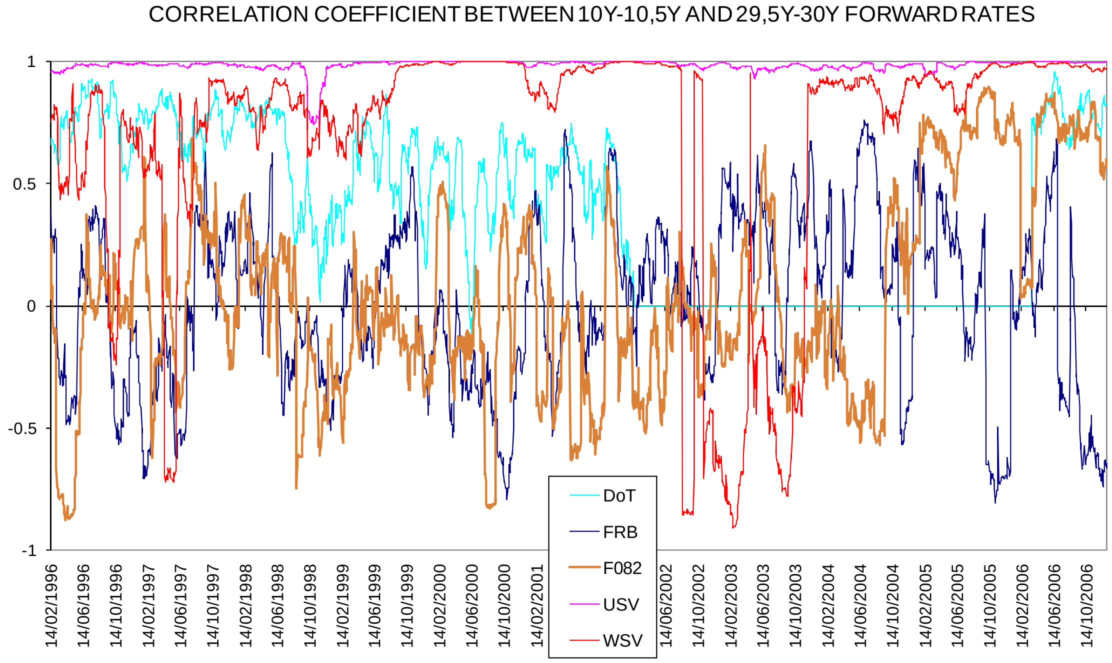


Table 1.- Summary statistics of TSIR estimations using Svensson (1994)

Panel A. Sum of squared residuals (SSR)

This table reports the yearly averages of the sum of squared residuals (SSR) from the daily estimation of the term structure of interest rates obtained by applying unweighted (USV) and weighted (WSV) versions of Svensson (1994) to different data sets from GovPx asset prices.

	(A) Full sample (bills, notes & bonds)		(B) Full sample excluding on-the-run and 1st off-the-run		(C) Mimicking FRB (excl. bills, on-the-run, 1st off-the-run)		(D) Mimicking DoT (only on-the-run) (11 maturities)*	
Year	USV	WSV	USV	WSV	USV	WSV	USV	WSV
1996	14.2	14.8	5.5	5.8	5.4	6.1	3.6	3.9
1997	9.8	10.2	4.8	5.0	4.7	5.5	2.1	2.4
1998	35.5	38.6	15.5	17.3	14.7	18.3	8.6	9.7
1999	70.0	78.1	33.0	38.2	32.8	36.7	16.7	18.3
2000	33.6	38.1	16.0	16.1	6.9	9.6	9.3	11.2
2001	43.5	45.9	21.0	22.6	20.7	24.1	10.2	10.5
2002	62.3	64.6	29.5	32.1	28.4	33.8	14.5	15.1
2003	41.9	38.3	23.0	23.9	21.7	24.3	9.4	9.6
2004	27.0	27.4	15.1	15.0	13.4	15.5	5.3	5.4
2005	15.2	16.3	8.1	9.2	8.0	9.4	3.0	3.2
2006	13.1	13.8	8.1	8.9	8.1	9.1	2.4	2.5
Avg	33.3	35.1	16.3	17.6	15.0	17.5	7.7	8.3

Note: all the samples exclude "when-issued" and cash management transactions, trades and quotes related to callable bonds and TIPS, and outliers (usual filters are applied).

* Four maturities of most recently auctioned bills (4-, 13-, 26-, and 52-week), six maturities of just-issued bonds and notes (2-, 3-, 5-, 7-, 10-, and 30-year), plus the composite rate in the 20-year maturity range.

Panel B. Composition of the data set

This table shows the average number of observations per day, the number of trading days in the year and the average maturity of the longest bond included in the daily estimation.

	# obs. per day				Days per	Maturity longest
Year	(A)	(B)	(C)	(D)	year	bond
1996	151.6	137.6	115.2	11.0	259	29.8
1997	150.1	136.2	114.0	11.0	261	29.8
1998	146.7	132.7	109.4	11.0	261	29.7
1999	134.4	120.4	96.8	11.0	261	29.8
2000	120.0	106.1	84.6	11.0	260	29.8
2001	114.6	100.6	79.6	11.0	258	29.5
2002	113.3	99.3	76.7	11.0	261	28.6
2003	115.2	101.2	78.6	11.0	261	27.6
2004	123.8	109.8	87.2	11.0	262	26.6
2005	129.6	115.7	93.7	11.0	260	25.6
2006	157.0	143.4	122.0	11.0	260	28.6
Avg	132.4	118.4	96.1	11.0	260	28.7

Panel C. Summary of estimated main parameters

(A) Full sample (bills, notes & bonds)

	USV						WSV					
	β_0	β_1	β_2	τ_1	β_3	τ_2	β_0	β_1	β_2	τ_1	β_3	τ_2
1996	7.21	-0.27	-0.08	0.73	0.12	1.48	7.27	-0.21	-0.04	1.06	0.09	1.96
1997	6.94	-0.21	-0.07	0.69	0.10	1.51	6.98	-0.18	-0.03	0.97	0.08	2.09
1998	6.00	-0.54	0.14	0.96	0.26	1.47	6.08	-0.25	0.01	1.13	0.12	1.76
1999	6.34	-1.08	0.45	0.83	0.53	0.95	6.49	-0.72	0.31	1.51	0.35	1.58
2000	6.03	-0.19	0.11	0.74	0.09	0.91	6.10	-0.75	0.37	0.85	0.37	0.88
2001	6.13	-1.02	0.41	0.95	0.49	1.00	6.20	-0.71	0.30	1.48	0.34	1.49
2002	6.31	-0.08	-0.03	1.34	0.02	1.31	6.36	-0.71	0.28	1.45	0.33	1.45
2003	6.23	-0.10	-0.03	1.72	0.03	1.72	6.11	-0.69	0.30	2.46	0.32	2.36
2004	6.14	-0.08	-0.02	2.00	0.02	2.00	6.21	-0.69	0.25	1.70	0.32	1.80
2005	5.01	-0.84	0.36	1.30	0.41	1.35	5.03	-0.57	0.21	1.18	0.28	1.35
2006	5.10	-0.90	0.41	1.13	0.45	1.17	5.09	-0.19	0.03	0.79	0.09	1.17
96/06	6.13	-0.48	0.15	1.13	0.23	1.35	6.18	-0.52	0.18	1.33	0.25	1.63

(B) Full sample excluding on-the-run and 1st off-the-run

	USV						WSV					
	β_0	β_1	β_2	τ_1	β_3	τ_2	β_0	β_1	β_2	τ_1	β_3	τ_2
1996	7.24	-0.26	-0.09	0.73	0.12	1.53	7.28	-0.22	-0.05	1.00	0.10	1.79
1997	6.95	-0.21	-0.07	0.70	0.10	1.51	6.98	-0.19	-0.04	0.92	0.08	1.68
1998	6.02	-0.76	0.32	1.33	0.37	1.42	6.09	-0.25	0.03	1.28	0.12	1.78
1999	6.37	-1.13	0.51	0.89	0.55	0.93	6.50	-0.73	0.32	1.43	0.35	1.50
2000	6.10	0.03	-0.19	0.36	0.10	1.53	6.12	-0.74	0.36	0.85	0.37	0.88
2001	6.15	-1.18	0.50	0.95	0.58	0.99	6.21	-0.76	0.33	1.45	0.37	1.46
2002	6.34	-1.16	0.44	0.96	0.55	1.02	6.40	-0.71	0.28	1.39	0.33	1.41
2003	6.28	-0.10	-0.03	1.80	0.02	1.80	6.34	-0.42	0.12	1.81	0.18	1.86
2004	6.19	-0.08	-0.02	2.07	0.02	2.07	6.24	-0.68	0.24	1.73	0.31	1.82
2005	5.05	-0.92	0.39	1.23	0.45	1.31	5.06	-0.47	0.15	1.18	0.23	1.40
2006	5.13	-0.88	0.40	1.21	0.44	1.25	5.11	-0.15	0.00	0.77	0.07	1.22
96/06	6.17	-0.60	0.20	1.11	0.30	1.40	6.21	-0.48	0.16	1.25	0.23	1.53

(C) Mimicking FRB (excl. bills, on-the-run, 1st off-the-run)

	USV						WSV					
	β_0	β_1	β_2	τ_1	β_3	τ_2	β_0	β_1	β_2	τ_1	β_3	τ_2
1996	7.24	-0.27	-0.21	0.58	0.14	1.44	7.32	-0.91	0.41	1.95	0.44	2.01
1997	6.95	-0.20	-0.19	0.54	0.11	1.43	7.03	-0.91	0.42	1.94	0.45	1.99
1998	6.02	-0.22	-0.38	0.49	0.17	1.36	6.14	-0.91	0.41	2.05	0.45	2.11
1999	6.36	-1.17	0.52	0.85	0.57	0.89	6.46	-1.01	0.46	1.42	0.49	1.47
2000	6.03	-0.76	0.23	2.28	0.44	6.24	5.98	-0.08	-0.02	2.16	0.06	4.57
2001	6.08	-1.05	0.42	1.06	0.50	1.11	6.25	-0.91	0.40	1.56	0.44	1.58
2002	6.33	-0.71	-0.18	0.61	0.45	1.23	6.46	-0.92	0.38	1.52	0.44	1.55
2003	6.27	-0.73	-0.33	0.61	0.35	1.27	6.36	-0.92	0.38	1.90	0.43	1.91
2004	6.17	-0.86	0.02	0.91	0.41	1.35	6.27	-0.93	0.38	1.82	0.44	1.87
2005	5.05	-1.03	0.44	1.21	0.50	1.28	5.13	-0.91	0.40	1.54	0.45	1.61
2006	5.13	-0.98	0.45	1.20	0.49	1.24	5.20	-1.02	0.48	1.70	0.51	1.73
96/06	6.15	-0.72	0.07	0.94	0.38	1.71	6.24	-0.86	0.37	1.78	0.42	2.04

(D) Mimicking DoT (only on-the-run) (11 maturities)

	USV						WSV					
	β_0	β_1	β_2	τ_1	β_3	τ_2	β_0	β_1	β_2	τ_1	β_3	τ_2
1996	7.01	-0.27	-0.11	0.49	0.13	1.12	7.09	-0.21	-0.03	0.84	0.09	1.48
1997	6.84	-0.20	-0.09	0.61	0.09	1.42	6.87	-0.18	-0.03	0.87	0.08	1.59
1998	5.86	-0.96	0.42	1.22	0.47	1.29	5.92	-0.26	0.02	1.04	0.12	1.48
1999	6.13	-1.13	0.52	0.89	0.55	0.91	6.28	-0.70	0.30	1.33	0.34	1.40
2000	5.84	0.04	0.01	0.73	-0.02	0.96	5.95	-0.73	0.37	0.78	0.36	0.80
2001	6.05	-0.89	0.37	1.15	0.43	1.19	6.07	-0.69	0.30	1.55	0.33	1.56
2002	6.13	-1.09	0.43	1.10	0.52	1.15	6.16	-0.71	0.29	1.43	0.33	1.43
2003	6.04	-0.10	-0.03	1.72	0.02	1.72	6.09	-0.67	0.25	1.78	0.31	1.79
2004	6.00	-0.17	0.02	1.94	0.06	1.95	6.06	-0.67	0.24	1.75	0.31	1.84
2005	4.92	-0.91	0.39	1.21	0.45	1.28	4.95	-0.47	0.15	1.14	0.22	1.36
2006	4.97	-1.30	0.62	0.92	0.65	1.05	4.98	-0.13	-0.01	0.64	0.06	1.09
96/06	5.98	-0.63	0.23	1.09	0.31	1.28	6.04	-0.49	0.17	1.20	0.23	1.44

Table 2.- Summary statistics of VTS estimations

Panel A: Historical Volatility from standard deviations

	Mean					Standard deviation				
Maturity	DoT	F082	FRB	USV	WSV	DoT	F082	FRB	USV	WSV
1 month	n/a	n/a	0.5824	0.5700	0.5483	n/a	n/a	1.7041	0.5466	0.1718
3 month	0.1900	0.2210	0.2354	0.3856	0.3587	0.1232	0.1392	0.2031	0.3278	0.1210
6 month	0.1811	0.2118	0.1829	0.2795	0.2415	0.1161	0.1391	0.1307	0.2076	0.1360
1 year	0.2219	0.2787	0.2388	0.2519	0.2490	0.1578	0.2324	0.1879	0.1861	0.1881
1.5 year	n/a	n/a	0.2607	0.2679	0.2728	n/a	n/a	0.1990	0.1989	0.2026
2 years	0.2680	0.2829	0.2623	0.2747	0.2784	0.1857	0.1994	0.1867	0.1951	0.1941
3 years	0.2528	0.2601	0.2481	0.2657	0.2663	0.1558	0.1608	0.1500	0.1652	0.1622
5 years	0.2225	0.2341	0.2165	0.2306	0.2304	0.1061	0.1137	0.0999	0.1100	0.1127
7 years	0.2022	n/a	0.1943	0.2012	0.2030	0.0805	n/a	0.0744	0.0796	0.0819
10 years	0.1836	0.1987	0.1729	0.1733	0.1759	0.0630	0.0683	0.0548	0.0562	0.0582
15 years	n/a	0.1752	0.1531	0.1523	0.1560	n/a	0.0527	0.0397	0.0402	0.0428
20 years	0.1488	0.1617	0.1394	0.1439	0.1473	0.0392	0.0436	0.0333	0.0346	0.0341
25 years	n/a	n/a	0.1311	0.1398	0.1477	n/a	n/a	0.0317	0.0324	0.0357
30 years	0.1326	0.1583	0.1355	0.1375	0.1552	0.0360	0.0482	0.0399	0.0314	0.0525

Panel B. Conditional volatility from EGARCH(1,1)

	Mean					Standard deviation				
Maturity	DoT	F082	FRB	USV	WSV	DoT	F082	FRB	USV	WSV
1 month	n/a	n/a	0.3960	0.3100	0.5049	n/a	n/a	0.9042	0.2841	0.1471
3 month	0.1864	0.1881	0.1887	0.2253	0.3043	0.1205	0.1187	0.1593	0.1831	0.1016
6 month	0.1835	0.1766	0.1829	0.1996	0.1913	0.1176	0.1165	0.1304	0.1449	0.1140
1 year	0.2284	0.2459	0.2498	0.2354	0.2360	0.1624	0.2036	0.1966	0.1736	0.1828
1.5 year	n/a	n/a	0.2732	0.2609	0.2662	n/a	n/a	0.2085	0.1936	0.2026
2 years	0.2781	0.2672	0.2738	0.2688	0.2726	0.1928	0.1873	0.1947	0.1908	0.1945
3 years	0.2654	0.2485	0.2602	0.2628	0.2617	0.1632	0.1531	0.1570	0.1632	0.1638
5 years	0.2325	0.2251	0.2266	0.2320	0.2285	0.1102	0.1091	0.1039	0.1104	0.1157
7 years	0.2113	n/a	0.2020	0.2022	0.2008	0.0837	n/a	0.0769	0.0797	0.0847
10 years	0.1909	0.1846	0.1790	0.1727	0.1732	0.0653	0.0635	0.0564	0.0559	0.0607
15 years	n/a	0.1658	0.1570	0.1509	0.1523	n/a	0.0498	0.0407	0.0398	0.0449
20 years	0.1520	0.1475	0.1425	0.1421	0.1440	0.0400	0.0400	0.0340	0.0341	0.0365
25 years	n/a	n/a	0.1332	0.1382	0.1394	n/a	n/a	0.0322	0.0320	0.0366
30 years	0.1380	0.1475	0.1384	0.1360	0.1395	0.0373	0.0448	0.0408	0.0310	0.0499

Department of Treasury (DoT), Bloomberg (F082), Federal Reserve Board (FRB) and our estimates of unweighted (USV) and weighted (WSV) Svensson models from the GovPx bond dataset.

Table 3.- Sign test of equal volatility estimates and the sample data

Table shows for each pair of data sets which one produces a statistically significant **higher** volatility

Panel A. Differences between data sets. Historical volatility (Standard Deviation)										
Maturity	DoT-F082	DoT-FRB	DoT-USV	DoT-WSV	F082-FRB	F082-USV	F082-WSV	FRB-USV	FRB-WSV	USV-WSV
1 month	n/a	n/a	n/a	n/a	n/a	n/a	n/a	USV^{a}	WSV^{a}	WSV^{a}
3 month	F082^{b}		USV^{a}	WSV^{a}		USV^{a}	WSV^{a}	USV^{a}	WSV^{a}	WSV^{a}
6 month	F082^{b}		USV^{a}	WSV^{a}		USV^{a}	WSV^{a}	USV^{a}	WSV^{a}	
1 year	F082^{a}		USV^{a}	WSV^{a}	F082^{c}			USV^{a}	WSV^{a}	
1.5 year	n/a	n/a	n/a	n/a	n/a	n/a	n/a		WSV^{b}	WSV^{b}
2 years			USV^{a}	WSV^{c}			WSV^{c}	USV^{c}	WSV^{a}	WSV^{a}
3 years	F082^{a}		USV^{a}	WSV^{a}	F082^{a}	USV^{b}	WSV^{a}	USV^{a}	WSV^{a}	WSV^{b}
5 years	F082^{a}	DoT^{b}	USV^{a}	WSV^{a}	F082^{a}			USV^{a}	WSV^{a}	
7 years	F082^{a}	DoT^{a}			n/a	n/a	n/a	USV^{a}	WSV^{a}	
10 years	F082^{a}	DoT^{a}	DoT^{a}	DoT^{a}	F082^{a}	F082^{a}	F082^{a}		WSV^{b}	WSV^{a}
15 years	n/a	n/a	n/a	n/a	F082^{a}	F082^{a}	F082^{a}		WSV^{b}	WSV^{a}
20 years	F082^{a}	DoT^{a}	DoT^{a}	DoT^{c}	F082^{a}	F082^{a}	F082^{a}	USV^{a}	WSV^{a}	WSV^{a}
25 years	n/a	n/a	n/a	n/a	n/a	n/a	n/a	USV^{a}	WSV^{a}	WSV^{a}
30 years	F082^{a}	DoT^{b}	DoT^{c}		F082^{a}	F082^{a}		USV^{a}	WSV^{a}	WSV^{a}

Panel B. Differences between data sets. Conditional volatility (E-GARCH)										
Maturity	DoT-F082	DoT-FRB	DoT-USV	DoT-WSV	F082-FRB	F082-USV	F082-WSV	FRB-USV	FRB-WSV	USV-WSV
1 month	n/a	n/a	n/a	n/a	n/a	N/A	N/A		WSV^{a}	WSV^{a}
3 month		DoT^{c}	USV^{a}	WSV^{a}			WSV^{a}	USV^{b}	WSV^{a}	WSV^{a}
6 month	DoT^{c}	DoT^{a}				USV^{b}	WSV^{b}		WSV^{b}	WSV^{c}
1 year								RF^{b}		
1.5 year	n/a	n/a	n/a	n/a	n/a	N/A	N/A	RF^{a}		WSV^{b}
2 years	DoT^{a}	DoT^{a}	DoT^{a}				WSV^{b}	RF^{a}		WSV^{a}
3 years	DoT^{a}		DoT^{a}		RF^{a}	USV^{a}	WSV^{a}			
5 years	DoT^{b}	DoT^{b}				USV^{a}	WSV^{b}			
7 years	DoT^{a}	DoT^{a}	DoT^{a}	DoT^{a}	n/a	N/A	N/A			
10 years	DoT^{a}	DoT^{a}	DoT^{a}	DoT^{a}	F082^{c}	F082^{b}		RF^{a}	RF^{c}	
15 years	n/a	n/a	n/a	n/a	F082^{b}	F082^{a}		RF^{a}		
20 years	DoT^{a}	DoT^{a}	DoT^{a}							WSV^{c}
25 years	n/a	n/a	n/a	n/a	n/a	N/A	N/A	USV^{a}	WSV^{a}	
30 years		DoT^{b}	DoT^{a}	DoT^{b}						

Panel C. Differences between volatility estimation method (same data set)					
Maturity	DoT	F082	FRB	USV	WSV
1 month	n/a	n/a	StDv^{b}	StDv^{a}	StDv^{a}
3 month	StDv^{a}	StDv^{b}	StDv^{b}	StDv^{a}	StDv^{a}
6 month	EGARCH^{a}	StDv^{a}		StDv^{a}	StDv^{a}
1 year	EGARCH^{a}	StDv^{a}		StDv^{a}	StDv^{a}
1.5 year	n/a	n/a		StDv^{a}	
2 years	EGARCH^{a}	StDv^{a}		StDv^{a}	
3 years	EGARCH^{a}	StDv^{a}	EGARCH^{a}	StDv^{a}	
5 years	EGARCH^{a}	StDv^{a}	EGARCH^{a}	EGARCH^{b}	
7 years	EGARCH^{a}	n/a	EGARCH^{a}	EGARCH^{b}	
10 years	EGARCH^{a}	StDv^{a}	EGARCH^{a}		
15 years	n/a	StDv^{a}	EGARCH^{a}	StDv^{a}	
20 years	EGARCH^{a}	StDv^{a}	EGARCH^{a}	StDv^{a}	
25 years	n/a	n/a	EGARCH^{a}	StDv^{a}	StDv^{a}
30 years	EGARCH^{a}	StDv^{a}	EGARCH^{a}	StDv^{a}	StDv^{a}

Sign test of equal volatility estimates from the five considered datasets: Department of Treasury (DoT), Bloomberg (F082), Federal Reserve Board (FRB) and our estimates of unweighted (USV) and weighted (WSV) Svensson models from the GovPx bond dataset. StDv is Standard Deviation.
Note: ^{a} $p<0.01$; ^{b} $p<0.05$; ^{c} $p<0.1$

Table 4.- Test of equal correlation coefficients

Table shows for each pair of data sets which one produces a statistically significant **higher** correlation coefficient between daily changes of pairs of 6-month spot rates and forward rates

Maturity	DoT-F082	DoT-FRB	DoT-USV	DoT-WSV	F082-FRB	F082-USV	F082-WSV	FRB-USV	FRB-WSV	USV-WSV
$R_{0.5}$-$F_{1,1.5}$	DoT[a]	FRB[a]	DoT[a]	DoT[a]	FRB[a]			FRB[a]	FRB[a]	
$R_{0.5}$-$F_{2,2.5}$	DoT[b]	FRB[a]			FRB[a]	USV[a]	WSV[a]	FRB[a]	FRB[a]	WSV[b]
$R_{0.5}$-$F_{5,5.5}$	DoT[a]	FRB[a]			FRB[a]	USV[a]		FRB[a]	FRB[a]	USV[b]
$R_{0.5}$-$F_{10,10.5}$	DoT[a]	DoT[b]	DoT[a]	DoT[a]	FRB[b]				FRB[b]	USV[b]
$R_{0.5}$-$F_{29.5,30}$	DoT[a]		USV[a]	WSV[a]	FRB[b]	USV[a]	WSV[a]	USV[a]	WSV[a]	
$F_{1,1.5}$-$F_{5,5.5}$	DoT[a]	FRB[a]	USV[a]		FRB[a]	USV[a]	WSV[a]		FRB[a]	USV[a]
$F_{1,1.5}$-$F_{10,10.5}$	DoT[a]	DoT[a]	DoT[a]	DoT[a]	FRB[a]	USV[a]	WSV[a]			
$F_{1,1.5}$-$F_{29.5,30}$	DoT[a]		USV[a]		FRB[a]	USV[a]	WSV[a]	USV[a]	WSV[a]	USV[b]
$F_{2,2.5}$-$F_{5,5.5}$	DoT[b]	FRB[a]	USV[a]		FRB[a]	USV[a]	WSV[b]		FRB[a]	USV[a]
$F_{2,2.5}$-$F_{10,10.5}$	DoT[a]	DoT[a]	DoT[a]	DoT[a]		USV[a]		USV[a]		
$F_{2,2.5}$-$F_{29.5,30}$	DoT[a]		USV[a]	WSV[a]		USV[a]	WSV[a]	USV[a]	WSV[a]	USV[a]
$F_{5,5.5}$-$F_{10,10.5}$	DoT[a]	DoT[a]		DoT[a]	FRB[a]	USV[a]		USV[b]	FRB[a]	USV[a]
$F_{5,5.5}$-$F_{29.5,30}$	DoT[a]	DoT[a]	USV[a]	DoT[c]	FRB[a]	USV[a]	WSV[a]	USV[a]	WSV[a]	USV[a]
$F_{10,10.5}$-$F_{29.5,30}$	DoT[a]	DoT[a]	USV[a]	WSV[a]		USV[a]	WSV[a]	USV[a]	WSV[a]	USV[a]

Sign test of equal correlation coefficient estimates from the five considered datasets: Department of Treasury (DoT), Bloomberg (F082), Federal Reserve Board (FRB) and our estimates of unweighted (USV) and weighted (WSV) Svensson models from the GovPx bond dataset. StDv is Standard Deviation.
Note: [a] $p<0.01$; [b] $p<0.05$; [c] $p<0.1$

Table 5. Comparison between spot interest rate data sets (Standard deviation volatilities)

Call prices and call deltas writing in theoretical callable bonds obtained from BDT estimations using zero coupon rates and volatilities from the five considered datasets: Department of Treasury (DoT), Bloomberg (F082), Federal Reserve Board (FRB) and our estimates of unweighted (USV) and weighted (WSV) Svensson models from the GovPx bond dataset.
The call price is the difference between a straight Treasury bond price and a callable Treasury bond price which includes one or two at par call options. Both securities are 7% semiannual coupon bonds. The call delta is the ratio between the change in price of the call option and the change in price of the underlying bond. For each model, we obtain the average value of call prices and call deltas for the 33 BDT estimations. These averages are compared to the average values for the 5 datasets. Sample period: first working day of March, July, and November from 1996 to 2006.

Term to			Gap between each dataset and the average of the 5 datasets						
Opt1	Opt2	Maturity			DoT	F082	FRB	USV	WSV
0.25	-	1.25	bond	%Var./Avg.	0.05%	-0.01%	-0.03%	-0.02%	0.01%
			call	%Var./Avg.	2.10%	-0.64%	-0.27%	-0.89%	-0.30%
			call	$Spread/Avg	$0.06	$-0.02	$-0.01	$-0.02	$-0.01
			delta	%Var./Avg.	0.14%	-0.08%	0.09%	-0.07%	-0.08%
0.5	-	5.5	bond	%Var./Avg.	0.13%	0.07%	-0.06%	-0.11%	-0.03%
			call	%Var./Avg.	1.46%	1.05%	-0.42%	-1.42%	-0.67%
			call	$Spread/Avg	$0.13	$0.10	$-0.04	$-0.13	$-0.06
			delta	%Var./Avg.	1.38%	-0.84%	1.65%	-1.12%	-1.08%
1	2	3	bond	%Var./Avg.	0.02%	-0.03%	-0.06%	0.02%	0.04%
			call	%Var./Avg.	-0.98%	-0.28%	-0.89%	1.04%	1.11%
			call	$Spread/Avg	$-0.04	$-0.01	$-0.04	$0.05	$0.05
			delta	%Var./Avg.	-0.40%	0.57%	1.53%	-0.81%	-0.90%
5	-	10	bond	%Var./Avg.	0.77%	0.79%	-0.54%	-0.57%	-0.45%
			call	%Var./Avg.	13.22%	15.56%	-9.26%	-11.14%	-8.38%
			call	$Spread/Avg	$0.76	$0.90	$-0.54	$-0.64	$-0.48
			delta	%Var./Avg.	6.00%	11.40%	-6.10%	-6.24%	-5.07%
10	20	30	bond	%Var./Avg.	1.42%	1.70%	0.07%	-1.53%	-1.65%
			call	%Var./Avg.	6.64%	13.92%	-0.89%	-10.37%	-9.31%
	(1)		call	$Spread/Avg	$0.74	$1.56	$-0.10	$-1.16	$-1.04
			delta	%Var./Avg.	3.78%	10.47%	1.21%	-6.91%	-8.55%
20	25	30	bond	%Var./Avg.	1.42%	1.70%	0.07%	-1.53%	-1.65%
			call	%Var./Avg.	12.18%	20.78%	-2.29%	-18.63%	-12.03%
	(1)		call	$Spread/Avg	$0.75	$1.28	$-0.14	$-1.15	$-0.74
			delta	%Var./Avg.	6.96%	16.86%	0.80%	-14.29%	-10.32%

Note: "bond" is the price of a 7% semiannual coupon straight Treasury bond, "call" is the price of a call option writing in a 7% semiannual coupon callable Treasury bond, "delta" is the call delta. For instance, the average call price using F082 dataset (Bloomberg) zero coupon rates and standard deviation volatilities for a Treasury bond with USD 100 principal, a remaining term to maturity of 30 years, and two call options at par on 20 and on 25 years is on average USD7.42 during the 33 BDT estimations. The average call price for the 5 models is USD6.15. Thus, the variation respect the average is (7.4241–6.1469)/6.1469 = 20.78% or USD1.28. In this sense, the Bloomberg dataset gives a call price 39.41% higher than the price obtained from the unweighted Svensson yield curve estimates obtained from the GovPX bond dataset, i.e. a difference of USD 2.42 per USD 100 principal.

(1) There are not zero coupon rates longer than 20 year in the DoT dataset from February 19, 2002 to March 23, 2006. Comparison between the 5 models for 30 year bonds excludes period from July 2002 to July 2005.

Table 6. Comparison between spot interest rate data sets (EGARCH volatilities)

Call prices and call deltas writing in theoretical callable bonds obtained from BDT estimations using zero coupon rates and volatilities from the five considered datasets: Department of Treasury (DoT), Bloomberg (F082), Federal Reserve Board (FRB) and our estimates of unweighted (USV) and weighted (WSV) Svensson models from the GovPx bond dataset.
The call price is the difference between a straight Treasury bond price and a callable Treasury bond price which includes one or two at par call options. Both securities are 7% semiannual coupon bonds. The call delta is the ratio between the change in price of the call option and the change in price of the underlying bond. For each model, we obtain the average value of call prices and call deltas for the 33 BDT estimations. These averages are compared to the average values for the 5 datasets. Sample period: first working day of March, July, and November from 1996 to 2006.

Term to			Gap between each dataset and the average of the 5 datasets						
Opt1	Opt2	Maturity			DoT	F082	FRB	USV	WSV
0.25	-	1.25	bond	%Var./Avg.	0.01%	0.00%	-0.03%	0.02%	0.01%
			call	%Var./Avg.	0.47%	-0.58%	-0.15%	0.48%	-0.22%
			call	$Spread/Avg	$0.01	$-0.02	$-0.00	$0.01	$-0.01
			delta	%Var./Avg.	0.04%	-0.08%	0.10%	0.02%	-0.07%
0.5	-	5.5	bond	%Var./Avg.	-0.01%	0.10%	0.02%	-0.09%	-0.05%
			call	%Var./Avg.	0.18%	1.21%	0.22%	-1.03%	-0.59%
			call	$Spread/Avg	$0.02	$0.11	$0.02	$-0.09	$-0.05
			delta	%Var./Avg.	-3.01%	-0.37%	-0.46%	-3.23%	7.07%
1	2	3	bond	%Var./Avg.	-0.06%	0.00%	-0.02%	0.04%	0.02%
			call	%Var./Avg.	-1.09%	-0.25%	0.13%	0.72%	0.49%
			call	$Spread/Avg	$-0.05	$-0.01	$0.01	$0.03	$0.02
			delta	%Var./Avg.	-1.54%	1.97%	0.49%	-0.55%	-0.38%
5	-	10	bond	%Var./Avg.	0.68%	0.79%	-0.49%	-0.55%	0.43%
			call	%Var./Avg.	12.90%	13.45%	-7.59%	-11.03%	-7.73%
			call	$Spread/Avg	$0.73	$0.76	$-0.43	$-0.63	$-0.44
			delta	%Var./Avg.	6.09%	9.87%	-6.69%	-6.16%	-3.11%
10	20	30	bond	%Var./Avg.	0.78%	1.45%	0.05%	-1.04%	-1.24%
			call	%Var./Avg.	6.31%	10.20%	4.20%	-7.07%	-13.64%
	(1)		call	$Spread/Avg	$0.66	$1.07	$0.44	$-0.74	$-1.43
			delta	%Var./Avg.	1.77%	7.53%	3.85%	-4.95%	-8.20%
20	25	30	bond	%Var./Avg.	0.78%	1.45%	0.05%	-1.04%	-1.24%
			call	%Var./Avg.	10.22%	21.56%	6.79%	-14.65%	-23.93%
	(1)		call	$Spread/Avg	$0.60	$1.26	$0.40	$-0.86	$-1.40
			delta	%Var./Avg.	6.07%	17.67%	5.95%	-11.14%	-18.56%

Note: "bond" is the price of a 7% semiannual coupon straight Treasury bond, "call" is the price of a call option writing in a 7% semiannual coupon callable Treasury bond, "delta" is the call delta. For instance, the average call price using F082 dataset (Bloomberg) zero coupon rates and EGARCH volatilities for a Treasury bond with USD 100 principal, a remaining term to maturity of 30 years, and two call options at par on 20 and 25 years is on average USD7.10 during the 33 BDT estimations. The average call price for the 5 models is USD5.84. Thus, the variation respect the average is (7.1025–5.8428)/5.8428 = 21.56% or USD1.26. In this sense, the Bloomberg dataset gives a call price 45.49% higher than the price obtained from the weighted Svensson yield curve estimates obtained from the GovPX bond dataset, i.e. a difference of USD 2.66 per USD 100 principal.
(1) There are not zero coupon rates longer than 20 year in the DoT dataset from February 19, 2002 to March 23, 2006. Comparison between the 5 models for 30 year bonds excludes period from July 2002 to July 2005.

Table 7. Replication of Table 1 "Means and Standard Deviations of Term Structure Variables" (Campbell, 1995)

Panel A. Department of Treasury (DoT)

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Excess return	0.448	0.455	0.471	0.487	0.528	0.585	0.713
	(0.204)	(0.210)	(0.237)	(0.344)	(0.757)	(1.638)	(3.795)
Change in yield	-0.018	-0.016	-0.015	-0.015	-0.033	-0.056	-0.081
	(0.244)	(0.205)	(0.202)	(0.220)	(0.271)	(0.292)	(0.267)
Yield spread	0.070	0.134	0.264	0.388	0.677	0.995	1.483
	(0.217)	(0.327)	(0.376)	(0.548)	(0.764)	(1.154)	(0.201)

Panel B. Bloomberg (F082)

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Excess return	0.439	0.450	0.471	0.490	0.523	0.587	0.715
	(0.204)	(0.210)	(0.239)	(0.356)	(0.754)	(1.637)	(3.717)
Change in yield	-0.023	-0.017	-0.013	-0.021	-0.035	-0.060	-0.084
	(0.237)	(0.196)	(0.198)	(0.228)	(0.270)	(0.292)	(0.261)
Yield spread	0.107	0.201	0.370	0.516	0.738	1.105	1.577
	(0.240)	(0.374)	(0.409)	(0.524)	(0.747)	(1.180)	(0.200)

Panel C. Federal Reserve Board (FRB)

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Excess return	0.466	0.468	0.475	0.492	0.526	0.586	0.743
	(0.207)	(0.212)	(0.236)	(0.348)	(0.744)	(1.627)	(3.682)
Change in yield	-0.008	-0.008	-0.011	-0.019	-0.035	-0.061	-0.086
	(0.216)	(0.199)	(0.192)	(0.221)	(0.266)	(0.290)	(0.259)
Yield spread	0.011	0.026	0.081	0.211	0.441	0.775	1.463
	(0.135)	(0.254)	(0.377)	(0.543)	(0.816)	(1.281)	(0.204)

Panel D. Unweighted Svensson model (USV) from the GovPx bond dataset

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Excess return	0.448	0.452	0.466	0.489	0.524	0.585	0.745
	(0.199)	(0.204)	(0.233)	(0.339)	(0.742)	(1.673)	(3.571)
Change in yield	0.000	-0.001	-0.006	-0.017	-0.038	-0.060	-0.087
	(0.311)	(0.272)	(0.219)	(0.216)	(0.265)	(0.298)	(0.251)
Yield spread	0.035	0.072	0.182	0.362	0.598	0.945	1.643
	(0.149)	(0.270)	(0.399)	(0.540)	(0.784)	(1.238)	(0.199)

Panel E. Weighted Svensson model (WSV) from the GovPx bond dataset

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Excess return	0.436	0.444	0.462	0.487	0.524	0.585	0.739
	(0.201)	(0.206)	(0.231)	(0.345)	(0.745)	(1.646)	(3.620)
Change in yield	-0.030	-0.023	-0.013	-0.014	-0.038	-0.062	-0.084
	(0.239)	(0.215)	(0.194)	(0.220)	(0.267)	(0.294)	(0.255)
Yield spread	0.066	0.126	0.275	0.480	0.724	1.063	1.756
	(0.114)	(0.228)	(0.356)	(0.513)	(0.779)	(1.240)	(0.199)

Panel F. Original Campbell's Table 1

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Excess return	0.379	0.553	0.829	0.862	0.621	0.475	-0.234
	(0.640)	(1.219)	(2.950)	(6.203)	(11.29)	(19.32)	(36.77)
Change in yield	0.014	0.014	0.014	0.014	0.014	0.014	0.013
	(0.591)	(0.575)	(0.569)	(0.546)	(0.486)	(0.408)	(0.307)
Yield spread	0.196	0.324	0.569	0.761	0.948	1.141	1.358
	(0.210)	(0.301)	(0.437)	(0.594)	(0.799)	(1.013)	(1.234)

Campbell footnote: "*Source: Author's calculations using estimated monthly zero-coupon yields, 1952-1991, from McCulloch and Kwon (1993). The data are measured monthly, but expressed in annualized percentage points. Each row shows the mean of the variable, with the standard deviation below in parentheses. Excess returns and yield spreads are measured relative to 1-month Treasury bill rates.*"

Table 8. Replication of Table 2 “Regression Coefficients” (Campbell, 1995)

Panel A. Department of Treasury (DoT)

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Short-run changes	1.086	0.660	0.778	1.297	0.182	-1.192	-2.050
In long yields	(0.014)	(0.014)	(0.023)	(0.048)	(0.087)	(0.137)	(0.210)
Long-run changes	3.112	1.494	1.224	1.442	1.035	0.918	-0.124
In short yields	(0.036)	(0.015)	(0.010)	(0.013)	(0.019)	(0.019)	(0.022)

Panel B. Bloomberg (F082)

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Short-run changes	0.915	0.592	0.507	0.976	0.081	-1.607	-1.806
In long yields	(0.012)	(0.012)	(0.020)	(0.046)	(0.091)	(0.140)	(0.202)
Long-run changes	2.578	1.328	1.046	1.147	0.952	0.967	0.000
In short yields	(0.032)	(0.013)	(0.008)	(0.012)	(0.019)	(0.018)	(0.030)

Panel C. Federal Reserve Board (FRB)

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Short-run changes	1.661	1.614	1.556	1.399	0.457	-0.907	-1.777
In long yields	(0.018)	(0.019)	(0.026)	(0.048)	(0.086)	(0.128)	(0.184)
Long-run changes	3.962	2.191	1.545	1.539	1.278	1.056	0.199
In short yields	(0.041)	(0.018)	(0.011)	(0.014)	(0.019)	(0.018)	(0.016)

Panel D. Unweighted Svensson model (USV) from the GovPx bond dataset

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Short-run changes	1.224	1.092	0.793	0.490	0.073	-0.692	-1.268
In long yields	(0.026)	(0.027)	(0.030)	(0.045)	(0.087)	(0.137)	(0.185)
Long-run changes	3.436	2.109	1.242	1.298	1.059	1.112	0.016
In short yields	(0.061)	(0.027)	(0.016)	(0.016)	(0.021)	(0.017)	(0.030)

Panel E. Weighted Svensson model (WSV) from the GovPx bond dataset

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Short-run changes	2.448	2.276	1.880	1.375	0.387	-1.102	-1.870
In long yields	(0.023)	(0.023)	(0.029)	(0.051)	(0.091)	(0.136)	(0.187)
Long-run changes	5.764	3.003	1.703	1.498	1.213	1.003	0.113
In short yields	(0.052)	(0.021)	(0.013)	(0.015)	(0.020)	(0.018)	(0.013)

Panel F. Original Campbell’s Table 1

	Long bond maturities (months)						
Variable	2	3	6	12	24	48	120
Short-run changes	0.019	-0.135	-0.842	-1.443	-1.432	-2.222	-4.102
In long yields	(0.194)	(0.285)	(0.444)	(0.598)	(0.996)	(1.451)	(2.083)
Long-run changes	0.51	0.473	0.301	0.253	0.341	0.435	1.311
In short yields	(0.097)	(0.149)	(0.147)	(0.210)	(0.221)	(0.398)	(0.120)

Campbell footnote: “*Source: Author's calculations using estimated monthly zero-coupon yields, 1952-1991, from McCulloch and Kwon (993). Each row shows a regression coefficient β, with the standard error below in parentheses. Each coefficient should be one if the expectations hypothesis holds. The regression in the first row is*

$$y_{m-1,t+1} - y_{m,t} = \alpha + \beta\left(y_{m,t} - y_{1,t}\right)/(m-1)$$

where m is long bond maturity in months. The regression in the second row is

$$\sum_{i=1}^{m-1} y_{1,t+i}/(m-1) - y_{1,t} = \alpha + \beta\left((m-1)/m\right)\left(y_{m,t} - y_{1,t}\right)$$

The standard error in the second row is corrected for serial correlation in the error term of the regression.”